**Math 821 Problem Set #1**
**Posted: Friday 1/28/11**
**Due date: Monday 2/7/11**

(Note: Some of these problems are proven as theorems in Munkres, but try to do them yourself.)

**Problem #1** Let $n$ and $m$ be nonnegative integers. Let $X = \mathbb{R}^n$ and $X' = \mathbb{R}^m$, both equipped with the standard topology. Prove that the product topology on $X \times X'$ is exactly the standard topology on $\mathbb{R}^{n+m}$.

**Problem #2** Let $(X, \mathscr{T})$ and $(X', \mathscr{T}')$ be topological spaces, let $\mathscr{B}'$ be a basis for the topology $\mathscr{T}'$, and let $f : X \to X'$ be a function. Prove that $f$ is continuous if and only if $f^{-1}(U)$ is open for all $U \in \mathscr{B}'$.

**Problem #3** Let $X$ be a path-connected topological space. Prove that $X$ is connected. (Recall that the converse is not true — the topologists' sine curve is a counterexample.)

**Problem #4** Let $\Gamma$ be a finite graph. (That is, $\Gamma$, consists of finitely many copies of the closed interval $I$, with some of the endpoints identified. More precisely, $\Gamma = X/\sim$, where $X$ is the disjoint union of finitely many copies $[a_1, b_1], \ldots, [a_n, b_n]$ of the closed unit interval $I$, and $\sim$ is an equivalence relation in which every point $p \notin \{a_1, b_1, \ldots, a_n, b_n\}$ induces a singleton equivalence class.)

Prove that if $\Gamma$ is connected, then it is path-connected. (In fact, this is true not merely for graphs, but for all finite cell complexes.)

**Problem #5** Let $X$ and $Y$ be topological spaces and let $f : X \twoheadrightarrow Y$ be a continuous function that is onto. Prove that if $X$ is connected, then so is $Y$.

**Problem #6** Let $X$ and $Y$ be topological spaces and let $f : X \twoheadrightarrow Y$ be a continuous function that is onto. Prove that if $X$ is compact, then so is $Y$.

**Problem #7** Let $n \geq 0$ be an integer, and let $X_n$ be the space you get by taking a strip of paper, twisting it $n$ times, and gluing the ends together. (So $X_0$ is a cylinder and $X_1$ is the Möbius strip.) For which pairs $n, m$ are $X_n$ and $X_m$ homeomorphic?