Axioms and theorems for plane geometry (Short Version)

Basic axioms and theorems

Axiom 1. *If A, B are distinct points, then there is exactly one line containing both A and B.*

Axiom 2. $AB = BA$.

Axiom 3. $AB = 0$ *iff* $A = B$.

Axiom 4. *If point C is between points A and B, then $AC + BC = AB$.*

Axiom 5. *(The triangle inequality) If C is not between A and B, then $AC + BC > AB$.*

Axiom 6. *Part (a): $m(\angle BAC) = 0°$ iff B, A, C are collinear and A is not between B and C. Part (b): $m(\angle BAC) = 180°$ iff B, A, C are collinear and A is between B and C.*

Axiom 7. *Whenever two lines meet to make four angles, the measures of those four angles add up to $360°$.*

Axiom 8. *Suppose that A, B, C are collinear points, with B between A and C, and that X is not collinear with A, B and C. Then $m(\angle AXB) + m(\angle BXC) = m(\angle AXC)$. Moreover, $m(\angle ABX) + m(\angle XBC) = m(\angle ABC)$.*

Axiom 9. *Equals can be substituted for equals.*

Axiom 10. *Given a point P and a line ℓ, there is exactly one line through P parallel to ℓ.*

Axiom 11. *If ℓ and ℓ' are parallel lines and m is a line that meets them both, then alternate interior angles have equal measure, as do corresponding angles.*

Axiom 12. *For any positive whole number n, and distinct points A, B, there is some C between A, B such that $n \cdot AC = AB$.*

Axiom 13. *For any positive whole number n and angle $\angle ABC$, there is a point D between A and C such that $n \cdot m(\angle ABD) = m(\angle ABC)$.*

Theorem 1. *All right angles have the same measure, namely $90°$.*

Theorem 2. *Every line segment $\overline{AB}$ has exactly one midpoint.*

Theorem 3. *Every angle $\angle BAC$ has exactly one bisector.*

Theorem 4. *If C is between A and B, then there is exactly one line ℓ passing through C that is perpendicular to $\overline{AB}$.*

Theorem 5. *Any two distinct lines intersect in at most one point.*

Theorem 6. *The sum of the interior angles of any triangle is $180°$. That is, if ΔABC is any triangle, then $m\angle ABC + m\angle BAC + m\angle ACB = 180°$.*

Theorem 7. *Suppose that two distinct lines m, m' both intersect a third line n. If alternate interior angles are equal, or if corresponding angles are equal then m and m' are parallel.*

Congruence and similarity

Axiom 14. *(SSS) Two triangles are congruent iff their corresponding sides are equal. That is, if ΔABC and $\Delta A'B'C'$ are two triangles such that $AB = A'B'$, $AC = A'C'$, and $BC = B'C'$, then $\Delta ABC \cong \Delta A'B'C'$.*

Axiom 15. *(AAA) Two triangles are similar iff their corresponding angles are equal. That is, if $m\angle BAC = m\angle B'A'C'$, $m\angle ABC = m\angle A'B'C'$, and $m\angle BCA = m\angle B'C'A'$, then $\Delta ABC \sim \Delta A'B'C'$.*

Theorem 8. *(ASA) Two triangles are congruent iff two pairs of corresponding angles, and the sides between them, are equal. That is, if* $m\angle BAC = m\angle B'A'C'$, $m\angle ABC = m\angle A'B'C'$, *and* $AB = A'B'$, *then* $\Delta ABC \cong \Delta A'B'C'$.

Theorem 9. *(SAS) Two triangles are congruent iff two pairs of corresponding sides, and the angles between those sides, are equal. That is, if* $AB = A'B'$, $AC = A'C'$, *and* $m\angle BAC = m\angle B'A'C'$, *then* $\Delta ABC \cong \Delta A'B'C'$.

Corollary 1. *Two triangles are similar iff two pairs of corresponding sides are proportional and the angles between those sides are equal.*

Other big theorems

Theorem 10. *(Thales' Theorem) The base angles of an iosceles triangle are equal. That is, if* $AB = AC$ *then* $\angle ABC \cong \angle ACB$.

Theorem 11. *Suppose that* $\overline{AB}$ *is a diameter of a circle centered at* O, *and that* C *is a point on the circle. Then* $m\angle ACB = 90^\circ$ *and* $m\angle BOC = 2m\angle BAC$.

Theorem 12 (Pythagorean Theorem/Gougu)**.** *If a right triangle has legs of lengths* a *and* b *and hypotenuse of length* c, *then* $a^2 + b^2 = c^2$.

Quadrilaterals

Theorem 13. *The angles of every quadrilateral add up to* 360°.

Theorem 14. *In a parallelogram* $PQRS$, *opposite sides and opposite angles are equal. That is, if* $\overleftrightarrow{PQ}$ *is parallel to* $\overleftrightarrow{RS}$ *and* $\overleftrightarrow{PS}$ *is parallel to* $\overleftrightarrow{QR}$, *then the following four things are true:* $PQ = RS$, $PS = RQ$, $\angle PQR \cong \angle RSP$, *and* $\angle QRS \cong \angle SPQ$.

Theorem 15. *The diagonals of every parallelogram bisect each other. That is, if* $PQRS$ *is any parallelogram, and* $X = \overline{PR} \cap \overline{QS}$ *is the point where its diagonals meet, then* $PX = RX$ *and* $QX = SX$.

Theorem 16. *The diagonals of parallelogram* $PQRS$ *meet at a right angle if and only if the parallelogram is a rhombus.*

Theorem 17. *The diagonals of a parallelogram are congruent to each other if and only if the parallelogram is a rectangle.*

Area

Axiom 16. *If two things are congruent, they have the same area.*

Axiom 17. *If* P *and* Q *are two sets, then* $\text{area}(P) + \text{area}(Q) = \text{area}(P \cup Q) + \text{area}(P \cap Q)$ *(provided that all these areas exist).*

Axiom 18. *A rectangle of length* a *and height* b *has area* ab.

Axiom 19. *If* $P \subseteq Q$, *then* $\text{area}(P) \leq \text{area}(Q)$.

Theorem 18. *A parallelogram with base* b *and height* h *has area* bh.

Theorem 19. *A triangle with base* b *and height* h *has area* $bh/2$.