One-Suit Whist

Whist (the ancestor of the modern game of bridge) is a fairly complicated card game, so here's a simpler version.

What you need: A deck of cards numbered 1, 2, ..., n. You can use any number of cards as long as it's even.

How to play: Deal half the deck to each player. Toss a coin to see who goes first. The first player plays a card and the other player tries to beat it with one of his or her cards. Whoever plays the higher card wins the trick, but has to play the first card to the next trick. The object is to win as many tricks as possible.

For example, if $n = 8$:

Round 1: Alice holds 1,3,5,7. Bob holds 2,4,6,8.
Alice loses the coin toss and has to play first.
Alice plays card 3.
Bob plays card 4 and wins the trick. **Score: Bob 1, Alice 0.**

Round 2: Alice holds 1,5,7. Bob holds 2,6,8.
Bob plays card 8.
Alice plays card 1. Bob wins the trick. **Score: Bob 2, Alice 0.**

Round 3: Alice holds 5,7. Bob holds 2,6.
Bob plays card 2.
Alice plays card 5 and wins the trick. **Score: Bob 2, Alice 1.**

Round 4: Alice holds 7. Bob holds 6.
Each player is down to one card, so they have no choice!
Alice wins the trick. **Final score: Bob 2, Alice 2.**

Research Questions

- Should you always win a trick if you can? Or is it sometimes better to lose a trick on purpose?

- Does the outcome of the coin toss necessarily matter?

- How much does the luck of the deal affect the outcome? That is, how strong does a hand have to be to take at least one trick? At least two tricks? At least half the tricks? All the tricks?

- All other things being equal, would you rather be dealt cards 7 and 4, or cards 6 and 5?

- Extend the game to multiple suits. For example, take a deck of cards and throw away the face cards, clubs and diamonds, so you're left with just the ace through ten of hearts (♡) and spades (♠). Separate the deck into suits and deal each player five cards of each suit. When it's your turn to play the first card to a trick, you can play any card in your hand. When you're going second, you have to play a card from the same suit (this is called "following suit").

  - Same questions as for one-suit whist. How strong does a hand have to be in order to take one/two/half/most/all the tricks?

  - Here's an additional wrinkle: If there's just one-suit whist, it's always better to win the coin toss and start by going second. (At least, it's never *worse* to win the coin toss.) Is this also true if there are multiple suits?

  - Suppose that we use two suits, and each player is dealt exactly one card of each rank. (For example: Alice gets $\heartsuit 1, 4, 5$ and $\spadesuit 2, 3, 6$; Bob gets $\heartsuit 2, 3, 6$ and $\spadesuit 1, 4, 5$.) Does it necessarily follow that the game must end in a draw with best play?

- Let's make it even more complicated. Shuffle the suits together and deal each player ten cards — but not necessarily the same number of cards of each suit! If you can't follow suit, then you can play any other card.

  - Version 1: If you can't follow suit, you simply lose the trick. Tough luck!
  - Version 2: Spades always beat hearts.

  If there are more than two suits, there are even more possibilities:

  - Version 3: Spades always beat hearts always beat diamonds always beat clubs.
  - Version 4: If you can't follow suit but you can play a spade, you win the trick. Otherwise, you lose it. (This is more like real whist or bridge — spades are called the trump suit.)