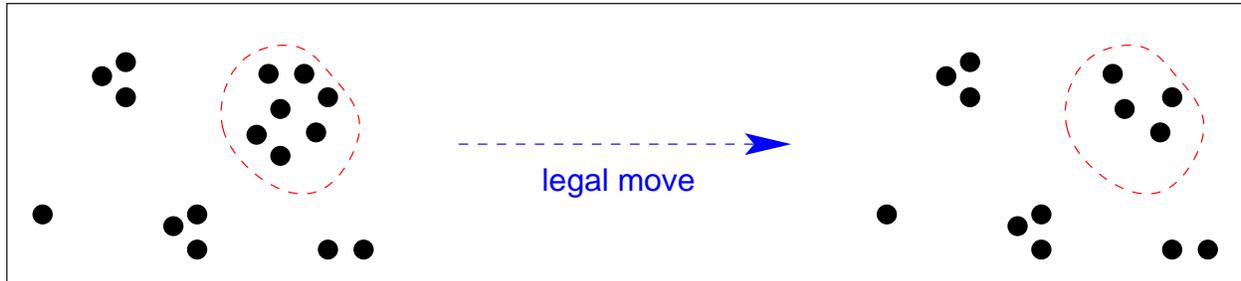


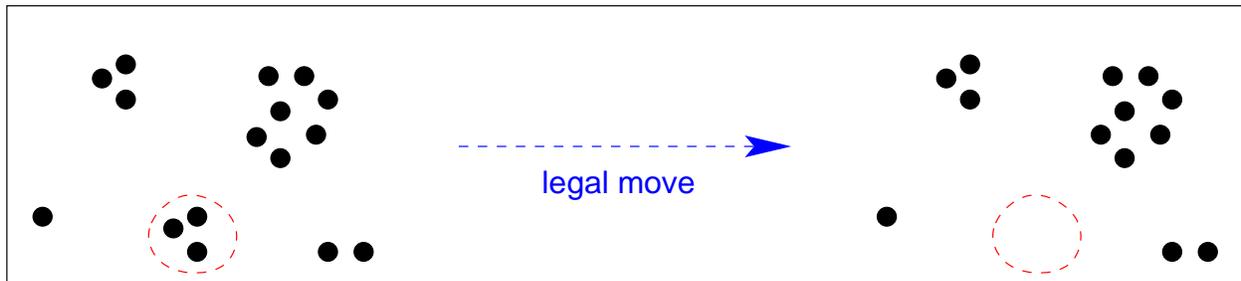
Nim

What you need: A pile of chips (or pennies, marbles, marshmallows, ...)

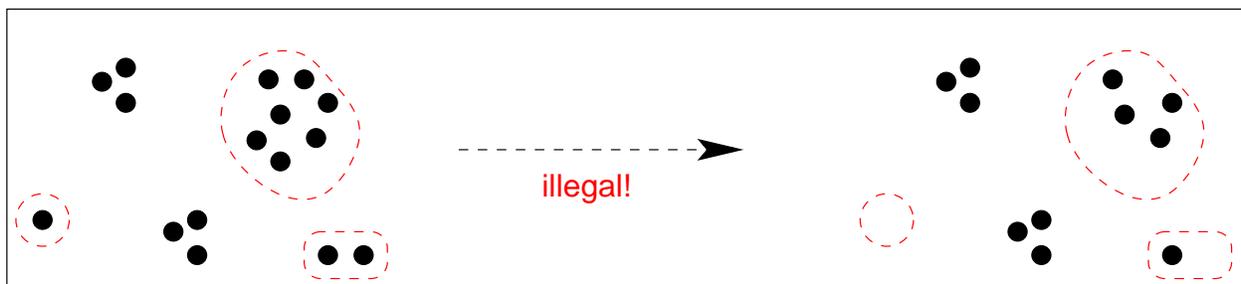
How to play: Divide the chips into piles. The two players take turns removing chips from the piles. On your turn, you may remove as many chips as you want, but only from one pile at a time.



It's okay to remove an entire pile:



But you can only take from one pile at a time.



Whoever takes the last chip wins.

Research Questions

The Strategy of Nim

- Is there always a winning strategy for one of the players?
- What happens when the game starts with one pile?
- What happens when the game starts with two piles?
- What happens when the game starts with three piles? Start with the following examples:
 - three piles that are all the same sizes (say 1 chip each, or 2 chips each, or...)
 - two piles of the same size and one pile containing a single chip
 - two piles of the same size and one pile containing fewer chips
 - three piles of all different sizes (say 1-2-3, or 1-2-4, or 1-3-4, or 2-3-5, or ...)
- What's the strategy when the game starts with an even number of piles, and every possible size occurs an even number of times? (E.g., 2-2-2-2-6-6-24-24-24-24-24-24-24-24-24-24-2010-2010.)
- Is there a winning strategy that works for any possible game?

Variants of Nim

- Whoever takes the last chip loses, instead of winning. (This is called *Misère Nim*.)
- If you've taken chips previously, you're allowed to put them back. (Again, you can only add to a single pile at a time.)
- Restrict the number of chips it's legal to remove at any one time (say, no more than 4).
- Brainstorm other variations!