Math 821, Spring 2014
Problem Set #1
Due date: Friday, January 31

As always, "space" means "topological space" and "map" means "continuous function".

Problem #1 Let X be a path-connected space. Prove that X is connected. (Recall that the converse is not true — the topologists' sine curve is a counterexample.)

Problem #2 Let Γ be a finite graph. (That is, Γ, consists of finitely many copies of the closed interval I, with some of the endpoints identified. More precisely, $\Gamma = X/\sim$, where X is the disjoint union of finitely many copies $[a_1, b_1], \ldots, [a_n, b_n]$ of the closed unit interval I, and $\sim$ is an equivalence relation in which every point $p \notin \{a_1, b_1, \ldots, a_n, b_n\}$ induces a singleton equivalence class.) Prove that if Γ is connected, then it is path-connected. (In fact, this is true not merely for graphs, but for all finite cell complexes.)

Problem #3 Let X and Y be spaces and let $f : X \twoheadrightarrow Y$ be a map that is onto. Prove that if X is compact, then so is Y.

Problem #4 [Hatcher p.18 #1] Construct an explicit deformation retraction of the torus with one point deleted onto a graph consisting of two circles intersecting in a point.

Problem #5 [Hatcher p.18 #3, more or less] (a) Show that the relation "X is homotopy equivalent to Y" is an equivalence relation.

(b) Fix spaces X, Y and let f, g be maps $X \to Y$. Show that the relation "f is homotopic to g" is an equivalence relation.

(c) Show that a map homotopic to a homotopy equivalence is a homotopy equivalence. More precisely: Let $f : X \to Y$ be a homotopy equivalence. Show that any map $g : X \to Y$ homotopic to f is a homotopy equivalence.

Problem #6 [Hatcher p.19 #14] Given nonnegative integers v, e, f with $v > 0$, $f > 0$, and $v - e + f = 2$, construct a cell structure on S^2 having v 0-cells, e 1-cells, and f 2-cells. (Do not use any facts about spanning trees or Euler characteristic.)