Instructions: Do all problems and typeset them in LaTeX. E-mail the PDF file to Jeremy at `jlmartin@ku.edu` under the filename `your-last-name.pdf` by **Friday, April 22, 5:00pm.** You are encouraged to use the LaTeX header file and to refer to Jeremy's lecture notes.

Problem #1 Complete the proof of Theorem 7.8 (connecting the deletion/contraction recurrence and the closed form of the Tutte polynomial) by showing that $\tilde{T}(G) = x \cdot T(G/e)$ if e is a bridge of G. (Hint: Mimic the proof for the loop case.)

Problem #2 Prove that $T(C_n; x, y) = x^{n-1} + \cdots + x^2 + x + y$ in two different ways:
(#2a) using the corank/nullity generating function;
(#2b) using the deletion/contraction recurrence and induction.

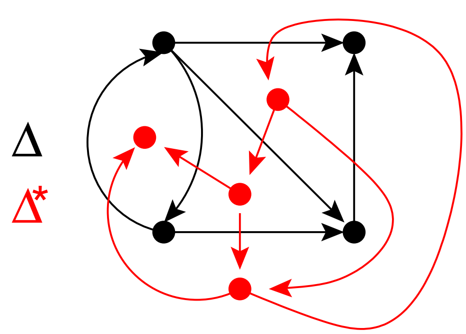
Then, convince yourself that the numbers of spanning trees, acyclic orientations and strong orientations of C_n, as well as its chromatic polynomial, are indeed given by the Tutte polynomial specializations we discussed in class. (For the chromatic polynomial, see Problem Set #5, Problem #3.)

Problem #3 Denote by $G^{(p)}$ the p^{th} *parallel extension of* G, i.e., the graph formed from G by replacing every edge of G with p parallel copies of itself. Find a formula for the Tutte polynomial of G in terms of that of G. (For example, if $G = K_2$, then $G^{(p)} \cong C_p^*$, the graph with two vertices and p parallel edges between them, so $T(G^{(p)}; x, y) = x + y + y^2 + \cdots + y^{p-1}$.) **Hint:** For $A \subseteq E(G^{(p)}$, let $\tilde{A}$ be the set of edges of G with at least one copy in A. How do $r(\tilde{A})$ and $r(A)$ compare? Write down the corank/nullity generating function for $T(G^{(p)}; x, y)$, then break it into pieces depending on what $\tilde{A}$ is.

Problem #4 Let Γ be a plane graph, which you can assume is connected.

(#4a) Prove that $(\Gamma - e)^* = \Gamma^*/e^*$ (provided that e is not a bridge) and that $(\Gamma/e)^* = \Gamma^* - e^*$ (provided that e is not a loop).
(#4b) We proved in class that $T(\Gamma; x, y) = T(\Gamma^*; y, x)$ by comparing the rank functions in Γ and Γ^*. Give another proof, using part (a) and the deletion/contraction recurrence for the Tutte polynomial.

Problem #5 Let Γ be a connected plane graph with no loops and let Δ be an orientation of Γ. The **dual orientation** Δ^* of Γ^* is defined as follows: if you stand at the tail vertex of $e \in E(G)$ and look toward the head, then the dual edge e^* points to the right.



Prove that Δ is an acyclic orientation of Γ if and only Δ^* is a strong orientation of Γ^*. (This explains why the numbers of acyclic and strong orientations are given by $T_G(2, 0)$ and $T_G(0, 2)$, respectively.)