Math 725, Spring 2010
Problem Set #5
Due date: Tuesday, March 30

#1. [West 3.3.16] Let G be a k-regular graph of even order* that remains connected when any $k-2$ edges are deleted. Prove that G has a perfect matching. (Hint: Modify the proof of Theorem 3.3.8 on p. 139 of West.)

#2. [West 4.1.9] For each choice of integers k, ℓ, m with $0 < k \leq \ell \leq m$, construct a simple graph G such that $\kappa(G) = k$, $\kappa'(G) = \ell$, and $\delta(G) = m$.

#3. [West 4.1.18] Let G be a triangle-free simple graph with minimum degree at least 3. Prove that if $n(G) \leq 11$, then G is 3-edge connected. Show that this inequality is sharp by constructing a 3-regular bipartite (simple) graph with 12 vertices that is not 3-edge connected. (Note the source of the problem!)

#4. [West 4.1.26, modified] Let G be a simple graph and let F be a nonempty set of edges in G.
(i) Prove that F is an edge cut if and only if F contains an even number of edges from every cycle in G.
(ii) Does your argument still work if G is not assumed to be simple?

#5. [West 4.2.20] Prove explicitly that for every n, the hypercube Q_n is n-connected. Do this explicitly by letting v, w be two arbitrary vertices of Q_n and constructing a family of n pairwise-internally-disjoint paths between v and w.

#6. [West 4.2.23, modified] Let G be an X, Y-bigraph. Let H be the graph obtained from G by adding two new vertices s, t, an edge sx for every $x \in X$, and an edge ty for every $y \in Y$.
(a) Prove that $\alpha'(G) = \lambda_H(s, t)$.
(b) Prove that $\beta(G) = \kappa_H(s, t)$.
(Therefore, the vertex version of Menger's Theorem implies the König-Egerváry Theorem.)

*Recall that "order" just means the number of vertices.