Math 725, Spring 2010
Problem Set #1
Due date: Tuesday, January 26

Note: Subsequent problem sets will contain fewer, and more challenging, problems than this one. Right now, the goal is to become comfortable with all the definitions, and to get some practice in the kinds of arguments necessary in graph theory.

#1. Problem 1.1.4 in West.

#2. [West 1.1.5, modified] Show that the statement "If G is a 2-regular simple graph, then G is a cycle" is false, by exhibiting an explicit counterexample. What additional condition on G makes the statement true?

#3. Problem 1.1.9 in West.

#4. A subgraph $H \subseteq G$ is called a *spanning subgraph* if $V(H) = V(G)$.

(a) Let G be an arbitrary graph. How many spanning subgraphs does G have? (Note: This is not the same as asking how many *isomorphism classes* of spanning subgraphs G has — that is a much harder problem.)

(b) Prove that G is connected if and only if it has a connected spanning subgraph.

(c) Use (b) to solve problem 1.1.10 in West. (Hint: If G is disconnected, what spanning subgraph must its complement have?)

#5. Let G be a graph and H a subgraph of G. Without using Theorem 1.2.18, prove that if H is not bipartite, then neither is G. Deduce the "only if" direction of Theorem 1.2.18 (i.e., *if* G is bipartite, *then* G has no odd cycle).

#6. Problem 1.2.1 in West.

#7. Problem 1.2.3 in West. if necessary.)

#8. Problem 1.2.5 in West.

#9. Problem 1.2.7 in West.

#10. Problem 1.2.22 in West.

Bonus problem: Let R_n be the graph on the bit strings of length n, in which two bit strings are adjacent if and only if they *agree* in exactly one bit. Show that $R_n \cong Q_n$ if and only if n is even.