MATH 724, Fall 2021
Homework #0
Your Name Here

Problem #0: Let $a(n)$ be the number of angels that can dance on the head of n pins. Prove that $a(n) = \frac{1}{n+1}\binom{2n}{n}n^{n-2}$.

Solution: We give a direct proof by contradiction and induction. The base case $n = 0$ is left to the reader. Assume inductively that the formula is true for all $m < n$. Define

$$\mathcal{S}_n = \{f : [n] \to [n] \mid f(i) > f(i-1) \ \forall i \leq \lfloor \frac{n}{2} \rfloor\}$$

or actually, come to think of it, define

$$\mathcal{S}_n = \left\{f : [n] \to [n] \mid f(i) > f(i-1) \ \forall i \leq \left\lfloor \frac{n}{2} \right\rfloor \right\}.$$

Then the desired result obviously follows as an immediate corollary. Also, do not write proofs like this.