Math 724, Fall 2017
Sample LaTeX File

For best results, download the .tex file and compare it to the .pdf output.

Theorem 0.1. *Let $n \in \mathbb{N}$. Then*

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}. \tag{1}$$

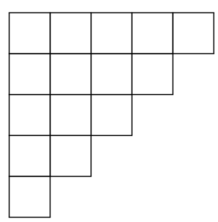
Proof. We proceed by induction on n. The base case $n = 1$ is obvious.[1]

For the inductive step, suppose that (1) holds for some $n \in \mathbb{N}$. We want to show that it holds for $n+1$, i.e., that $\sum_{k=1}^{n+1} k = (n+1)(n+2)/2$. To see this, note that

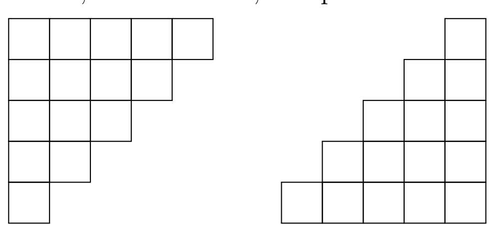
$$\begin{aligned}
\sum_{k=1}^{n+1} k &= \left(\sum_{k=1}^{n} k\right) + (n+1) \\
&= \frac{n(n+1)}{2} + (n+1) && \text{(by induction)} \\
&= \frac{n^2+n}{2} + \frac{2n+2}{2} \\
&= \frac{n^2+3n+2}{2} = \frac{(n+1)(n+2)}{2}
\end{aligned}$$

as desired. □

Alternate proof. Build a staircase of height n with k blocks in the k^{th} row, so that the total number of blocks it contains is $\sum_{k=1}^{n} k$. For example, if $n = 5$ then the staircase looks like this:



Now make a photocopy of the staircase, rotate it 180°, and put it next to the original staircase:



Push the two staircases together. They form a rectangle with height n and width $n+1$. By the Fundamental Theorem of Combinatorics[2], the number of squares in the rectangle is

$$n(n+1) = 2\sum_{k=1}^{n} k$$

which is equivalent to (1). □

[1] "Obvious" is one of my least favorite words — if you write that something is obvious, it had better be really, really obvious, like $1 = 1$ (as here)

[2] "If you count a set in two different ways, you get the same answer."