

Math 724, Fall 2017

Homework #4

Deadline: Friday, October 13, 5:00pm

Instructions: Typeset your solutions in LaTeX. Email your solutions to Jeremy (jlmartin@ku.edu) as a PDF file named with your last name and the problem set number (e.g., `Noether4.pdf`). Collaboration is encouraged, but each student must write up his or her solutions independently and acknowledge all collaborators.

(#1) Problem #128.

(#2) Problem #129.

(#3) Problem #141.

(#4) Problem #142. Implement your recurrence for (b) in Sage, and include the code you write in your submitted solutions. (Use the `verbatim` environment to get LaTeX to pay attention to line breaks and indentation.) The [Sage Vignette](#) on the course website (also available as [LaTeX source](#)) may be helpful; you may use or adapt the Sage code for Stirling numbers therein. You can certainly use your code to answer part (c).

(#5) Problem #149.

(#6) Problem #150. (Notice that this problem says “onto” where the previous one says “to.” So in #149 you are counting all functions; in #150 you are counting surjections.)

(#7) How many partitions $\lambda = (\lambda_1, \dots, \lambda_n)$ with n parts have the property that $1 \leq \lambda_k \leq n + 1 - k$ for all k ? (Hint: Write down all such partitions for $n = 1, 2, 3, 4$, and you will have a pretty good guess of what the answer is. Then find an appropriate bijection.)

(Note: By definition, a partition is a weakly decreasing sequence of positive integers: $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n > 0$. For instance, if $n = 3$, the sequence $\lambda = (1, 2, 1)$ is not a partition even though it satisfies the criterion $1 \leq \lambda_k \leq n + 1 - k$ for all k .)