

Here is how the method is supposed to work. Several of you found descriptions of the method online, including:

http://spiff.rit.edu/classes/phys311/extra/earth_radius/earth_rad.html

<http://www.darylsience.com/downloads/Db1Sunset.pdf>

Here's my explanation and figure.

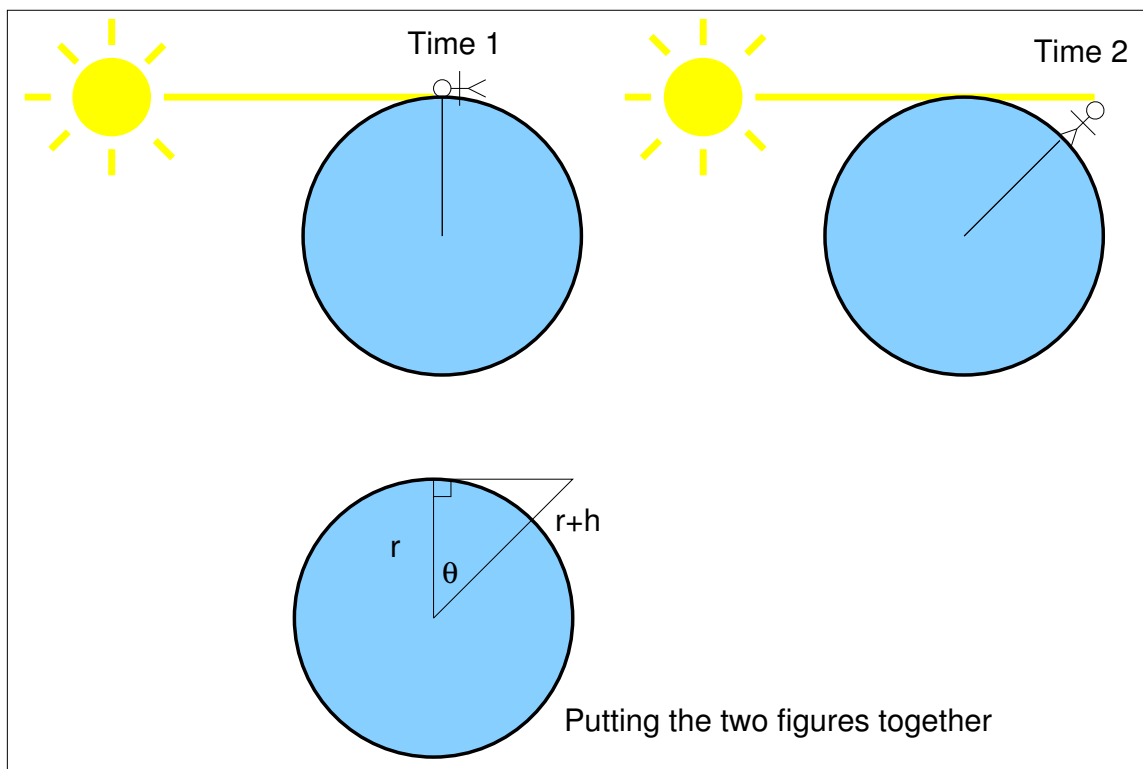
Start by lying down on the ground and watching the sunset. At the instant that the sun disappears over the horizon (Time 1), start your stopwatch. Then stand up, so that the sun becomes visible again. When the sun becomes invisible (Time 2), stop the stopwatch, and record the time T (let's say in seconds).

The "setting" of the sun is caused by the rotation of the earth on its axis. During sunset, the point that you are standing is rotating away from the sun so that the sun gradually becomes invisible. Therefore, the angle θ (measured in degrees) through which the earth has rotated is related to T by

$$\frac{T}{\text{number of seconds in a day}} = \frac{\theta}{360}. \quad (1)$$

If we take the number of seconds in a day as 86400 ($= 60 \text{ seconds/minute} \times 60 \text{ minutes/hour} \times 24 \text{ hours/day}$) then this equation becomes

$$\frac{T}{86400} = \frac{\theta}{360} \quad \text{or} \quad \theta = 240T. \quad (2)$$



Let r be the radius of the earth and let h be your height. Then the figure at the bottom gives

$$\cos \theta = \frac{r}{r+h}.$$

Note that h can be measured and θ can be calculated from T using (2). So we need to solve for r in terms of T and θ :

$$\begin{aligned}\cos \theta &= \frac{r}{r+h} \\ (r+h) \cos \theta &= r \\ r \cos \theta + h \cos \theta &= r \\ h \cos \theta &= r - r \cos \theta \\ \frac{h \cos \theta}{1 - \cos \theta} &= r\end{aligned}$$

Now plugging in (2) gives

$$r = \frac{h \cos(240T)}{1 - \cos(240T)}.$$

Here are some problems with this approach, including hidden assumptions.

First, h is not really your height, but rather the difference

$$\begin{aligned}&(\text{height of your eyes above the ground when you are standing up}) \\ &- (\text{height of your eyes above the ground when you are lying down})\end{aligned}$$

which is probably a few inches less than your actual height.

Second, T is likely to be very small and hard to measure accurately, and a small error may make a large distance in the final value of θ and thus r . One way of improving accuracy could be to have one person stand at the top of a tall tower and another at the bottom, and have each record when the sun disappears from view (synchronize your watches!)

(In all fairness, Eratosthenes' method suffers from similar problems with accuracy of measurement.)

Third, the number of seconds in a day is not really 86400 (it's close), so there is a little inaccuracy here.

But probably the biggest flaw in the procedure is that it assumes that θ is the actual angle through which the earth has rotated. This is only true if you're standing perpendicular to the axis of rotation — in other words, it only works if you're on the equator. (Thought experiment: We are assuming that the tilt of the earth does not change. Under that assumption, if you are standing on the North Pole, then the sun will never disappear from view (provided it was visible to begin with). A little more generally, this method will tend to overestimate θ in summer and underestimate it in winter.)

We should try this ourselves sometime when it stops snowing.