

**Math 223, Fall 2012**  
**Review Information for Final Exam**

---

**General Information**

**Time, date and place of exam: Monday, December 10, 7:30 AM – 10:00 AM, Snow 120** (the usual classroom). Try to arrive early so that we can start on time. (Yes, I know it's early in the morning!)

Bring pens or pencils, scratch paper, and a calculator. Check the batteries beforehand — two students may not share a calculator, and I will not have extras available. You don't need to bring a bluebook.

The exam is closed-book and closed-notes; however, a list of formulas will be provided to you (see next page).

The exam will be comprehensive, covering chapters 1, 2, 3, 6 and 7 of Colley. The focus will be on chapter 7.

**Office hours on Friday 12/7: 10:00-12:00 and 2:00-4:00, Snow 623.** I will also try to check e-mail regularly over the weekend ([jmartin@math.ku.edu](mailto:jmartin@math.ku.edu)) to answer questions.

Please check the KU website for updates in case of inclement weather. The Office of the Provost asks instructors to inform students that no final examination will be canceled or postponed to a later date because of a building evacuation; the complete policy is available here: <https://documents.ku.edu/policies/provost/FinalExamsEvacuating.htm>.

---

**Review Topics for Chapter 7**

Please see review sheets for the two midterm tests for topics covered in chapters 3–6, as well as suggested review problems.

In Chapter 7, you are responsible for the material in §7.1–7.3. Specific topics and suggested review problems

- Important surfaces to know how to parametrize: plane, disk, sphere, cylinder, cone (throughout chapter 7)
- Given a parametrized surface  $\mathbf{X} : D \subseteq \mathbb{R}^2 \rightarrow \mathbb{R}^3$ , find coordinate curves, tangent vectors, normal vectors and tangent planes, and express the surface as the graph of a function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  or as a level surface of a function  $f : \mathbb{R}^3 \rightarrow \mathbb{R}$  (§7.1: #1–19; §7.6: #1–8)
- Find the surface area of a parametrized surface or of the graph of a function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  (§7.1: #22–29)
- Set up and evaluate a scalar or vector surface integral along a parametrized surface in  $\mathbb{R}^3$  (§7.2: #1–27)
- Understand the interpretation of a vector surface integral in terms of flux
- Know what happens to a surface integral when the surface is reparametrized (§7.2: #4)
- Verify Stokes' Theorem for a given surface and vector field (§7.3: #1–5)
- Verify the Divergence Theorem (a.k.a. Gauss's Theorem) for a given closed surface and vector field (§7.3: #6–9)
- Applications of Stokes' Theorem and the Divergence Theorem, including replacing a difficult integral with easier ones (§7.3: #11–18, 20)

## Formulas that will be given to you on the exam

### Conversion between rectangular and spherical coordinates:

$$\begin{cases} x = \rho \sin \phi \cos \theta \\ y = \rho \sin \phi \sin \theta \\ z = \rho \cos \phi \end{cases} \quad \begin{cases} \rho^2 = x^2 + y^2 + z^2 \\ \tan \phi = \sqrt{x^2 + y^2}/z \\ \tan \theta = y/x \end{cases}$$

**Product and Quotient Rules:** If  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  and  $g : \mathbb{R}^n \rightarrow \mathbb{R}$  are differentiable at  $\mathbf{a} \in \mathbb{R}^n$ , then

$$D(fg)(\mathbf{a}) = g(\mathbf{a})Df(\mathbf{a}) + f(\mathbf{a})Dg(\mathbf{a})$$

$$D(f/g)(\mathbf{a}) = \frac{g(\mathbf{a})Df(\mathbf{a}) - f(\mathbf{a})Dg(\mathbf{a})}{g(\mathbf{a})^2} \quad (\text{provided that } g(\mathbf{a}) \neq 0)$$

**Chain Rule:** If  $g : \mathbb{R}^n \rightarrow \mathbb{R}^m$  and  $f : \mathbb{R}^m \rightarrow \mathbb{R}^k$  are functions such that  $g$  is differentiable at  $\mathbf{a} \in \mathbb{R}^n$ , and  $f$  is differentiable at  $g(\mathbf{a}) \in \mathbb{R}^m$ , then

$$D(f \circ g)(\mathbf{a}) = [Df(g(\mathbf{a}))] [Dg(\mathbf{a})]$$

### Curvature:

$$\kappa = \left\| \frac{d\mathbf{T}}{ds} \right\| = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|^3}$$

### Torsion:

$$\frac{d\mathbf{B}}{ds} = -\tau\mathbf{N}; \quad \tau = \frac{(\mathbf{v} \times \mathbf{a}) \cdot \mathbf{a}'}{\|\mathbf{v} \times \mathbf{a}\|^2}$$

### Line integrals:

$$\int_{\mathbf{x}} f \, ds = \int_a^b f(\mathbf{x}(t)) \|\mathbf{x}'(t)\| \, dt \quad \int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{x}(t)) \cdot \mathbf{x}'(t) \, dt$$

### Green's Theorem:

$$\oint_{\partial D} \mathbf{F} \cdot d\mathbf{s} = \iint_D (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dA \quad \text{or} \quad \oint_{\partial D} M \, dx + N \, dy = \iint_D (N_x - M_y) \, dx \, dy$$

### Surface integrals:

$$\iint_{\mathbf{X}} f \, dS = \iint_D f(\mathbf{X}(s, t)) \|\mathbf{N}(s, t)\| \, ds \, dt \quad \iint_{\mathbf{X}} \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F}(\mathbf{X}(s, t)) \cdot \mathbf{N}(s, t) \, ds \, dt$$

### Stokes' Theorem:

$$\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \oint_{\partial S} \mathbf{F} \cdot d\mathbf{s}$$

### Divergence Theorem:

$$\iiint_B (\nabla \cdot \mathbf{F}) \, dV = \iint_{\partial B} \mathbf{F} \cdot d\mathbf{S}$$