Math 223, Fall 2010
Review Information for Test #1

1. Logistics. The test will be in class on **Friday 10/1/10**. Wednesday's class will include some time for review. Bring a supply of loose-leaf paper to the test. You may bring a calculator.

2. Topics. You are responsible for material from lectures and homeworks through Friday, September 24 (essentially chapters 1–2 in Colley). Specific topics to know:

- Basic vector arithmetic (§1.1)
- Equations of lines in $\mathbb{R}^n$, both parametric and non-parametric (e.g., §1.2: 13–21, 24–28)
- Parametrizing curves such as circles, ellipses, and cycloids (§1.2: 37–39)
- Dot product, cross products, and determinants:
  - calculations and general properties (§1.3: 1–9, 17–19, 28; §1.4: 5–9, 27–33)
  - how to use them to find angles, distance, volume, and other geometric information (§1.3: 14–17, 23–27; §1.4: 11–17)
- Working with equations of planes, using different kinds of data (§1.5: 13–18, 30)
- Sketching level curves of a function $f : \mathbb{R}^2 \to \mathbb{R}$ and using them to describe the graph of f (§2.1: 10–19)
- Describing the level surfaces of a function $f : \mathbb{R}^3 \to \mathbb{R}$ (§2.1: 28–35)
- Evaluating limits of functions $f : X \subseteq \mathbb{R}^n \to \mathbb{R}$ (§2.2: 7–23, 28–33)
- Determining whether a function $f : X \subseteq \mathbb{R}^n \to \mathbb{R}$ is continuous (§2.2: 34–44)
- Finding the partial derivatives and the derivative matrix of a function $F : X \subseteq \mathbb{R}^n \to \mathbb{R}^m$ (§2.3: 1–25, 38)
- Finding an equation for the tangent space to a function $f : X \subseteq \mathbb{R}^n \to \mathbb{R}$ at a given point in X (§2.3: 29–34)
- Understanding what continuity and differentiability mean geometrically (§2.3: 50)
- Higher-order partial derivatives (§2.4: 9–19)
- Using the Chain Rule for calculations (§2.5: 1–4, 15–21) and short proofs (§2.5: 9–14, 22, 27–30)
- What the directional derivative means (§2.6: 1)
- Calculating directional derivatives (§2.6: 2–10)
- What the gradient tells you about directional derivatives (§2.6: 11–15)
- Finding the tangent space to an implicitly defined surface (§2.6: 16–39)

Obviously, doing all the problems listed above is impractical. In order to study for the test, do a few problems on each topic. If it seems easy, go on to the next one. If it seems hard, then do more problems on it!

Topics from Chapters 1 and 2 for which you are *not* reponsible:

- Symmetric form of the equation of a line in $\mathbb{R}^3$ (eqn. (7), p. 12)
- Torque, rotation, angular velocity (pp. 33–34)
- Standard bases for cylindrical and spherical coordinates (pp. 69–71)
- Classification of quadric surfaces (pp. 89–91) (that is, you don't have to know what "hyperboloid of one sheet", "hyperbolic paraboloid", etc., mean)
- Newton's method (pp. 130–137)
- The Implicit and Inverse Function Theorems (pp. 162–167)

3. Formulas. The following formulas will be provided to you on the test. You don't have to memorize them, but you do need to know how and when to use them and what the notation means. (If you are working on a review problems and you need a formula that is not on the list below, then that means that you need to know it.)

Projection: For all vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$,

$$\text{proj}_{\mathbf{a}} \mathbf{b} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}} \right) \mathbf{a}$$

Triangle inequality: For all vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$,

$$\|\mathbf{a} + \mathbf{b}\| \leq \|\mathbf{a}\| + \|\mathbf{b}\|$$

Conversion between rectangular and spherical coordinates:

$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases} \qquad \begin{cases} \rho^2 = x^2 + y^2 + z^2 \\ \tan\phi = \sqrt{x^2 + y^2}/z \\ \tan\theta = y/x \end{cases}$$

Product and Quotient Rules:

If $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}$ are differentiable at $\mathbf{a} \in \mathbb{R}^n$, then

$$D(fg)(\mathbf{a}) = g(\mathbf{a})Df(\mathbf{a}) + f(\mathbf{a})Dg(\mathbf{a}),$$

$$D(f/g)(\mathbf{a}) = \frac{g(\mathbf{a})Df(\mathbf{a}) - f(\mathbf{a})Dg(\mathbf{a})}{g(\mathbf{a})^2} \qquad \text{(provided that } g(\mathbf{a}) \neq 0\text{)}$$

Chain Rule:

If $g : \mathbb{R}^n \to \mathbb{R}^m$ and $f : \mathbb{R}^m \to \mathbb{R}^k$ are functions such that g is differentiable at $\mathbf{a} \in \mathbb{R}^n$, and f is differentiable at $g(\mathbf{a}) \in \mathbb{R}^m$, then

$$D(f \circ g)(\mathbf{a}) = \left[Df(g(\mathbf{a})) \right] \left[Dg(\mathbf{a}) \right]$$