Math 223, Fall 2010
Extra Credit Problem #1
Due date: Friday 8/27/10

Prove that

$$\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\|\|\mathbf{w}\| \cos\theta$$

for all $\mathbf{v}, \mathbf{w} \in \mathbb{R}^2$. Here, as always "prove" means "**come up with an argument that is always valid, no matter what vectors you plug in**"; it is not enough (it is never enough!) to merely give an example.

To do this, fill in the details of the following argument.

1. Draw $\mathbf{v}$ and $\mathbf{w}$ on a coordinate axis. Let α and β be the angles formed by $\mathbf{v}$ and $\mathbf{w}$ in standard position.

2. Express α and β in terms of the components v_1, v_2, w_1, w_2 of $\mathbf{v}$ and $\mathbf{w}$.

3. Next, express θ in terms of α and β.

4. Then, calculate $\cos\theta$ (you'll need some trigonometry here; in particular, you'll need to rewrite expressions like $\cos(\arctan z)$ without using trig functions).

5. Finally, show that the result is equal to $\dfrac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|}$.