Math 141 Honors Problems #5
Due date: Tuesday, 9/22/09

HP9 [4 points]

(a) Construct a function $p(x)$ with domain $\mathbb{R}$ that is differentiable but not second-differentiable. That is, $p'(x)$ is defined and continuous on $\mathbb{R}$, but $p''(x)$ has a discontinuity (say, at $x = 0$).

(b) Let n be an arbitrary positive integer. Construct a function $q(x)$ with domain $\mathbb{R}$ such that q is $(n-1)^{th}$-order differentiable but not n^{th}-order differentiable. That is,

$$q, \quad \frac{dq}{dx}, \quad \frac{d^2q}{dx^2}, \quad \ldots, \quad \frac{d^{n-1}q}{dx^{n-1}}$$

are all defined and continuous, but $\dfrac{d^nq}{dx^n}$ has a discontinuity (again, say, at $x = 0$).

(As always, the stipulation "let n be an arbitrary positive integer" doesn't mean "pick your favorite specific value of n"; it means "find a solution, expressed in terms of n, that is valid for all possible n".)

HP10 [1 point each]

Evaluate the following limits. You can use a table of values to estimate them if you want to, but to earn the point, your final answer should use precise tools such as the Limit Laws (see §2.3), continuity, and the Squeeze Theorem.

(10a) $\displaystyle\lim_{x\to 0} \frac{\sin x}{x + x^2}$

(10b) $\displaystyle\lim_{x\to 0} \frac{\sin^2(3x)}{x^2 \cos\, x}$

(10c) $\displaystyle\lim_{x\to 0} \frac{x - \tan\, x}{\sin\, x}$

(10d) $\displaystyle\lim_{\theta\to 0} \frac{\cos\theta - 1}{\sin^2\theta}$