Math 141 Honors Problems #10
Due date: Tuesday, 11/3/09

HP19 [3 points] First, read the (probably apocryphal) story about the ten-year-old Carl Friedrich Gauss and the sum $1 + 2 + 3 + \cdots + 99 + 100$ (type "Gauss 100" into Google, or see Appendix F).

A version of Gauss's proof using modern summation notation: If $S = \sum_{i=1}^{n} i$ then

$$2S = \left(\sum_{i=1}^{n} i\right) + \left(\sum_{i=1}^{n} (n - i + 1)\right)$$

$$= \sum_{i=1}^{n} (i + n - i + 1)$$

$$= \sum_{i=1}^{n} (n + 1)$$

$$= n(n + 1),$$

so $S = n(n + 1)/2$. (This is the same idea as the "staircase" picture proof from class on Monday 11/2.)

Using the same idea, prove the identity

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n + 1)^2}{4}.$$

You may use the identity

$$\sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}$$

in the course of the proof.

HP20 [4 points] Suppose that p is a positive real number. *Without using the Fundamental Theorem of Calculus*, evaluate

$$\int_0^1 p^x \, dx.$$

(Hint: Geometric series.)