Problem HP1: Here's the formula you were aiming for:

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}.$$

In English, "the sum of the squares of the numbers in the n^{th} row of Pascal's Triangle equals the middle number in the $(2n)^{th}$ row" — but the formula is a nice concise way to say that.

Here's the explanation I gave in class on Wednesday.

First, we know that $\binom{2n}{n}$ means the number of subsets of a set of $2n$ elements. In other words, this is the number of different ways to select an n-person subcommittees from a group of $2n$ people.

Here's another way to calculate that number of choices. First, let's arbitrarily take our set of $2n$ people, paint n of them crimson, and paint the other n people blue. Now, let's classify each possible n-person subcommittee by the number of blue people on it. That number can be anything from 0 to n; let's call it k.

Number of blue people	Number of crimson people	Number of possible subcommittees
0	n	$\binom{n}{0}\binom{n}{n}$
1	$n-1$	$\binom{n}{1}\binom{n}{n-1}$
2	$n-2$	$\binom{n}{2}\binom{n}{n-2}$
$\vdots$		
k	n-k	$\binom{n}{k}\binom{n}{n-k}$
$\vdots$		
$n-1$	1	$\binom{n}{n-1}\binom{n}{1}$
n	0	$\binom{n}{n}\binom{n}{0}$

Adding up the right-hand column is another way to count the total number of n-person committees. That gives

$$\binom{n}{0}\binom{n}{n} + \binom{n}{1}\binom{n}{n-1} + \cdots + \binom{n}{n-1}\binom{n}{1} + \binom{n}{0}\binom{n}{n} = \sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k} = \sum_{k=0}^{n} \binom{n}{k}^2$$

where the second equality uses the fact that $\binom{n}{k} = \binom{n}{n-k}$ for all n and k (this is the left-right symmetry of Pascal's Triangle).

To recap, we've counted the number of n-element subcommittees of a $(2n)$-element set in two ways. Both ways must yield the same answer: that is,

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}.$$

Problem HP2: As most of you noticed, as n gets larger and larger, the quantity $a(n) = s(n) - n!$ increases without bound, but on the other hand, the quantity $b(n) = s(n)/n!$ gets closer to 1. In other words,

$$\lim_{n\to\infty} a(n) \;=\; \lim_{n\to\infty} s(n) - n! \;=\; +\infty \qquad (A)$$

and

$$\lim_{n\to\infty} a(n) \;=\; \lim_{n\to\infty} \frac{s(n)}{n!} \;=\; 1. \qquad (B)$$

Many solvers drew the conclusion that it was therefore better to use $b(n)$ to measure the accuracy of the approximation, rather than $a(n)$. However, this conclusion doesn't necessarily follow; maybe the truth is that Stirling's formula is actually a lousy way of approximating $n!$, as suggested by equation (A). To put it another way, there's a distinction between the accuracy of $s(n)$ itself and the accuracy of our various means of testing its accuracy!

In fact, provided that $s(n)$ and $n!$ both increase without bound (as they certainly do), condition (A) is *logically stronger* than condition (B). (We'll be able to prove this shortly.) I would argue that $b(n)$ is a better measure of accuracy than $a(n)$, since it is essentially measuring the *percentage* by which $s(n)$ differs from $n!$; since the numbers are so large, we'd expect the difference to be very large even if $s(n)$ is only off by a tiny percentage. In other word, equation (B) says that $s(n)$ is an excellent approximation to $n!$ for large values of n, while equation (A) just says that it's maybe not super-double-plus-excellent.