Math 141 Homework #8
Due Tuesday, 10/2/07
Extra Problems

These problems are taken from the Math 121 sample midterm exam from Fall 2005.

Problem #1 What value of x is $f(x) = x^3 + \frac{1}{2}x^2 - 2x - 3$ decreasing most rapidly?

Problem #2 If c is a constant, then $\lim\limits_{h \to 0} \dfrac{e^{ch} - 1}{h}$ equals

(a) $\ln c$ (b) c (c) e^c (d) 0 (e) none of the above

Problem #3 Evaluate $\lim\limits_{h \to -4} \dfrac{h+4}{\sqrt{h+6} - \sqrt{2}}$ or explain why it does not exist.

Problem #4 Let $f(x) = \dfrac{x - \sqrt{3}}{x^2 - 3}$. Evaluate the following:

(#4a) $\lim\limits_{x \to 1} f(x)$
(#4b) $\lim\limits_{x \to 3} f(x)$
(#4c) $\lim\limits_{x \to \sqrt{3}} f(x)$
(#4d) $\lim\limits_{x \to -2} f(x)$

Problem #5 Suppose that f and g are functions such that f is continuous, $f(-1) = 3$, and $\lim\limits_{x \to -1} \dfrac{g(x)}{f(x)^2 + 1} = 8$. Find $\lim\limits_{x \to -1} g(x)$.

Problem #6 Suppose that $f(3) = 2$, $f'(3) = -1$, $g(3) = 3$, and $g'(3) = 5$. Find the following numbers: (i) $(fg)'(3)$; (ii) $(g/f)'(3)$; (iii) the derivative of $x^{-1}/f(x)$ at $x = 3$.

Problem #7 Let $f(x) = x^2 + 1$. Find every number a such that the line tangent to the graph of $f(x)$ at the point $(a, f(a))$ passes through the point $91, 0)$.

Problem #8 The position of a particle at time t is given by $s(t) = t^3 - 4t^2 + 3t$ for $t \geq 0$.

(#8a) When is the velocity equal to 6?
(#8b) When is the acceleration equal to 0?
(#8c) When does the particle reverse its direction of motion?

Problem #9 Let f be the function defined by $f(x) = 2x - 1$ for $x \geq 1$ and $f(x) = 3x - 2$ for $x < 1$. At $a = 1$, the function f is

(a) continuous
(b) discontinuous because $\lim_{x\to 1} f(x)$ does not exist as a real number
(c) discontinuous because $\lim_{x\to 1} f(x) \neq f(1)$
(d) none of the above

Problem #10 Find an equation for the tangent line to the parametric curve $(x, y) = (3\sin t,\ e^{2t})$ at the point $(0, 1)$.

Problem #11 Find an equation for the tangent line to the curve $3(x^2 + y^2)^2 = 14x^2 - y^2$ at the point $(\sqrt{2}, 1)$.

Problem #12 Calculate $\dfrac{d}{dx}\left[x^2 + 3)^{\sin x}\right]$.

Problem #13 The derivative of $f(x) = \cos(x^2)$ at $x = 0$ is given by the expression

(a) $\displaystyle\lim_{h\to 0} \frac{\cos(h^2) - \cos h}{h}$ (b) $\displaystyle\lim_{h\to 0} \frac{\cos(h^2) - 1}{h^2}$ (c) $\displaystyle\lim_{h\to 0} \frac{\cos(h^2) - 1}{h}$ (d) $\displaystyle\lim_{h\to 0} \frac{\cos h - 1}{h}$ (e) none of the above

Problem #14 For which value(s) of c is the function $f(x)$ defined below continuous everywhere?

$$f(x) = \begin{cases} c^2 x \text{ if } x \leq 1 \\ c + 6x \text{ if } x > 1 \end{cases}$$

Problem #15 Suppose that the tangent line to the graph of $f(x)$ at $(-1, 2)$ passes through the point $(1, 5)$. Find $f(-1)$ and $f'(-1)$.

Problem #16 Suppose that $f(3) = 2$ and $f'(3) = 5$. Find the derivative of $(x^2 + 1)^{f(x)}$ at $x = 3$.

Problem #17 Calculate the following limits, and for each one, draw a conclusion about an asymptote of some function.

(#17a) $\displaystyle\lim_{x\to-\infty} \frac{x^2 + \sqrt{3}x^3 + \sqrt{5}}{\sqrt{2} - 5x - \sqrt{2}x^3}$

(#17b) $\displaystyle\lim_{x\to\infty} \left(-2x + \sqrt{4x^2 - 3x + 1}\right)$

(#17c) $\displaystyle\lim_{x\to 3^-} \frac{x - 1}{(x-3)(x-4)}$

Problem #18 Let $f(x) = e^x$, $g(x) = x - 3$, and $h(x) = 5x$. Find the functions fg, $f \circ g$, and $f \circ g \circ h$.

Problem #19 Find a formula for the inverse of (i) the function $g(t) = e^{1-t} + 3$ with domain $(-\infty, \infty)$ and (ii) the function $f(x) = \ln(x^2 + x - 1$ with domain $(1, \infty)$.