

Problem: Three cousins (Ana, Belle and Chloe) are dividing an estate consisting of four items of furniture. The cousins' valuations of the items are shown in the following table. Determine a fair division using the Method of Sealed Bids.

Items \ Players	Ana	Belle	Chloe
Vanity	150	300	275
Desk	170	150	165
Tapestry	190	200	250
Dresser	510	250	540

Step 1: Find each player's **total** valuation of the complete set of goods to be divided. Divide by the number of players to determine each player's **fair share**.

Items \ Players	Ana	Belle	Chloe
Vanity	150	300	275
Desk	170	150	165
Tapestry	190	200	250
Dresser	510	250	540
Total	1020	900	1230
Fair share	340	300	410

For example, Ana's total is $150 + 170 + 160 + 510 = 990$, and her fair share is $990/3 = 330$.

Step 2: Determine who gets which item by awarding each item to the highest bidder. (It is possible for a player to get multiple items, or to get no items.)

Items \ Players	Ana	Belle	Chloe
Vanity	150	300*	275
Desk	170*	150	165
Tapestry	190	200	250*
Dresser	510	250	540*
Total	1020	900	1230
Fair share	340	300	410

For example, Ana gets the desk because her bid of \$170 was higher than Belle's \$150 or Chloe's \$165.

Step 3: Determine the total value of the items received by each player.

Items \ Players	Ana	Belle	Chloe
Vanity	150	300*	275
Desk	170*	150	165
Tapestry	190	200	250*
Dresser	510	250	540*
Total	1020	900	1230
Fair share	340	300	410
Item value	170	300	790

For example, Chloe got the tapestry and the dresser, so her item value is $250 + 540 = 790$.

Step 4: Subtract each player's item value from her fair share to determine the **first settlement**. If this number is positive, the player receives cash from the estate (since her item value was less than the fair share.) If this number is negative, the player must pay cash to the estate (since her item value was more than the fair share.)

Be careful about the signs!

Items \ Players	Ana	Belle	Chloe
Vanity	150	300*	275
Desk	170*	150	165
Tapestry	190	200	250*
Dresser	510	250	540*
Total	1020	900	1230
Fair share	340	300	410
Item value	170	300	790
First settlement	170	0	-380

For example, Ana's fair share was \$340 but she only received \$170 worth of items, so in order to make up a fair share she must receive $\$340 - \$170 = \$170$ from the estate.

Belle's fair share was \$300 and she received items worth exactly that. So she neither receives or owes any money at this stage.

Meanwhile, Chloe's fair share was \$410 but her item value was \$790, so in order to receive an exact fair share she must pay the estate \$380. Note that $410 - 790 = -380$; the minus sign indicates that she owes money.

Step 5: Add up the first-settlement numbers. If you have done everything correctly up to this point, the number you get will always be negative or zero (since more cash should have been paid to the estate than taken out of it). Take off the negative sign to get the amount of extra cash on hand. This is the **surplus**.

In this case the sum of the first-settlement numbers is

$$170 - 380 = -210$$

which tells you that the surplus is \$210.

Items \ Players	Ana	Belle	Chloe
Vanity	150	300*	275
Desk	170*	150	165
Tapestry	190	200	250*
Dresser	510	250	540*
Total	1020	900	1230
Fair share	340	300	410
Item value	170	300	790
First settlement	170	0	-380
Surplus	210		

Step 6: Divide the surplus by the number of players to find each player's **share of surplus**. Here, \$210 divided by 3 equals \$70.

Items \ Players	Ana	Belle	Chloe
Vanity	150	300*	275
Desk	170*	150	165
Tapestry	160	200	250*
Dresser	510	250	540*
Total	990	870	1230
Fair share	330	290	410
Item value	170	300	790
First settlement	170	0	-380
Surplus	210		
Share of surplus	70	70	70

Step 7: Add each player's first settlement to her share of the surplus share to obtain her **final settlement**.

As before, a plus sign indicates that the player receives money, while a minus sign indicates that the player owes money.

If you have done everything correctly, the final settlements should all add up to zero.

Items \ Players	Ana	Belle	Chloe
Vanity	150	300*	275
Desk	170*	150	165
Tapestry	160	200	250*
Dresser	510	250	540*
Total	990	870	1230
Fair share	330	290	410
Item value	170	300	790
First settlement	170	0	-380
Surplus	210		
Share of surplus	70	70	70
Final settlement	240	70	-310

In fact, $240 + 70 - 310 = 0$, so everything works out. Each player has now received a combination of goods and cash that are worth \$70 more than what she would have considered a fair share.