

# Quick Review of TSP Algorithms

- ▶ **Brute force:** Examine all  $(N - 1)!$  Hamilton circuits individually; choose the cheapest one.
- Finds the optimal answer but is very inefficient.
- ▶ **Nearest-Neighbor Algorithm (NNA):** Pick a reference vertex. At each step, walk to the nearest vertex not already visited.
- ▶ **Repetitive Nearest-Neighbor Algorithm (RNNA):** Perform the NNA from every possible reference vertex, obtaining  $N$  different Hamilton circuits. Choose the cheapest one.

# The Cheapest-Link Algorithm

The **Cheapest-Link Algorithm (CLA)** is a bit different.

Instead of starting at a reference vertex and moving to the nearest neighbor at each step, we “start in the middle.”

That is, if there is a cheap edge that you know you will want to use eventually — make sure you use it!

At each stage of the algorithm, pick the cheapest edge available, regardless of what its endpoints are.

Of course, all the edges you pick have to come together to form a circuit.

# The Cheapest-Link Algorithm

Here's a more precise description of the Cheapest-Link Algorithm.

# The Cheapest-Link Algorithm

Here's a more precise description of the Cheapest-Link Algorithm.

- ▶ Find the cheapest edge that you haven't already added.

# The Cheapest-Link Algorithm

Here's a more precise description of the Cheapest-Link Algorithm.

- ▶ Find the cheapest edge that you haven't already added.
- ▶ Add it to the list of edges to use.

# The Cheapest-Link Algorithm

Here's a more precise description of the Cheapest-Link Algorithm.

- ▶ Find the cheapest edge that you haven't already added.
- ▶ Add it to the list of edges to use.
- ▶ Keep doing this until you have a Hamilton circuit.

# The Cheapest-Link Algorithm

Here's a more precise description of the Cheapest-Link Algorithm.

- ▶ Find the cheapest edge that you haven't already added.
- ▶ Add it to the list of edges to use.
- ▶ Keep doing this until you have a Hamilton circuit.
- ▶ Make sure you add exactly two edges at each vertex. (In other words, don't put a third edge at a vertex, and don't close the circuit too early.)

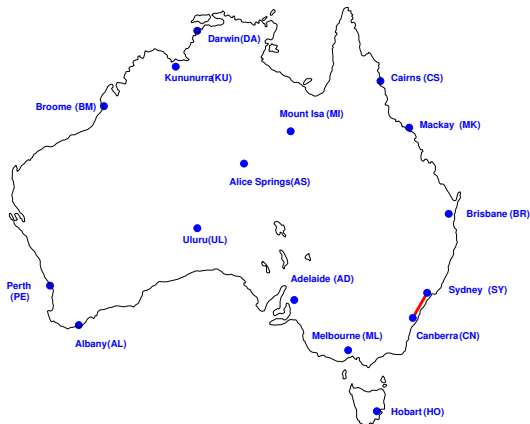
# The Cheapest-Link Algorithm

Here is an example of designing a tour of Australia using the Cheapest-Link Algorithm.

(Warning: The figure is not quite to scale!)

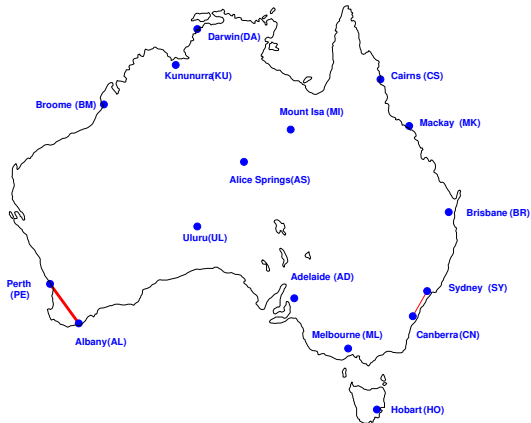


# The Cheapest-Link Algorithm



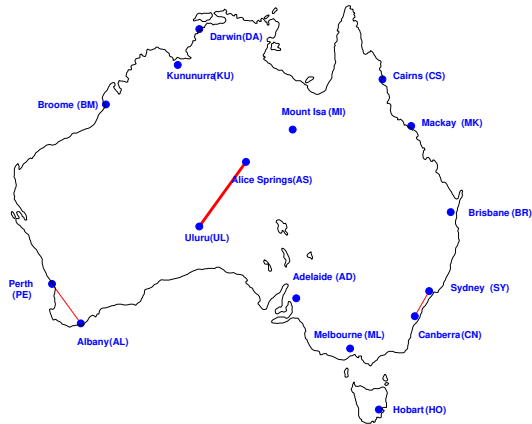
Start by finding the shortest edge (Sydney–Canberra).

# The Cheapest-Link Algorithm



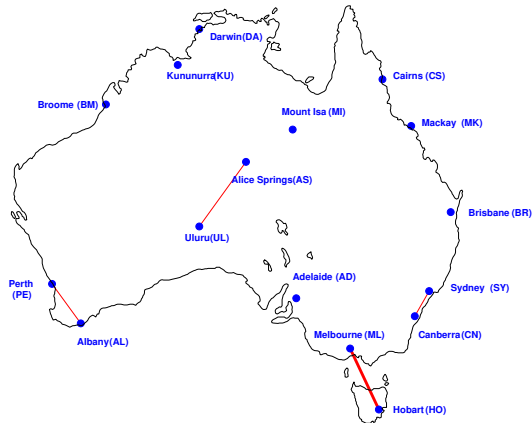
Now find the next shortest edge (Perth–Albany).

# The Cheapest-Link Algorithm



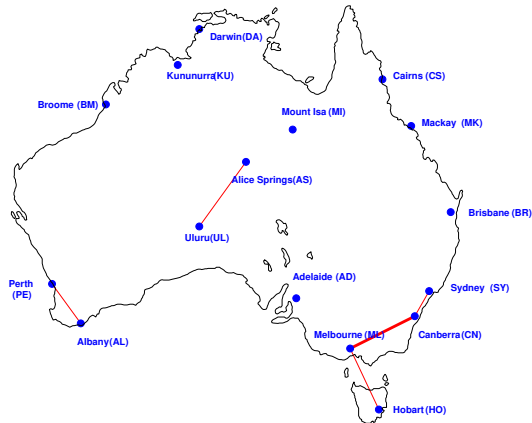
And the next shortest (Alice Springs–Uluru)...

# The Cheapest-Link Algorithm



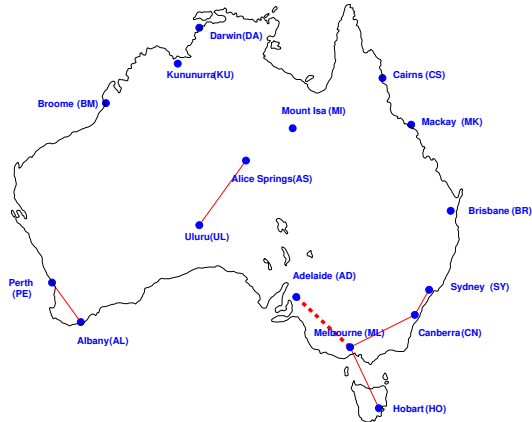
and the next shortest (Hobart–Melbourne)...

# The Cheapest-Link Algorithm



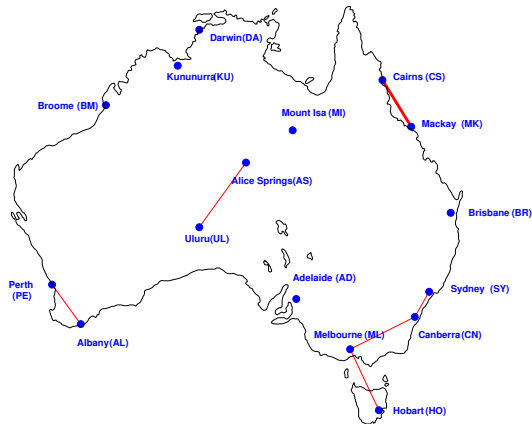
and the next shortest (Canberra-Melbourne).

# The Cheapest-Link Algorithm



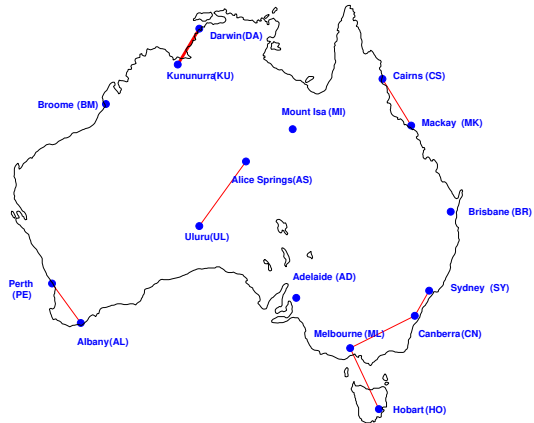
The next shortest edge is actually Adelaide–Melbourne, but we can't use it because Melbourne already has two edges.

# The Cheapest-Link Algorithm



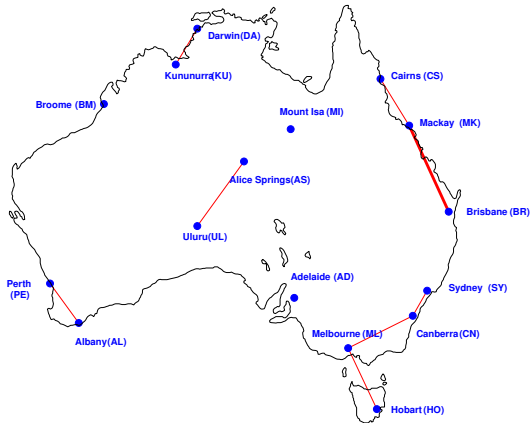
So instead we include the next shortest edge (Cairns-Mackay).

# The Cheapest-Link Algorithm

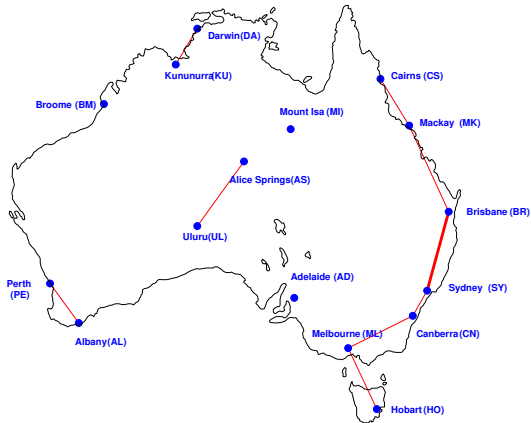




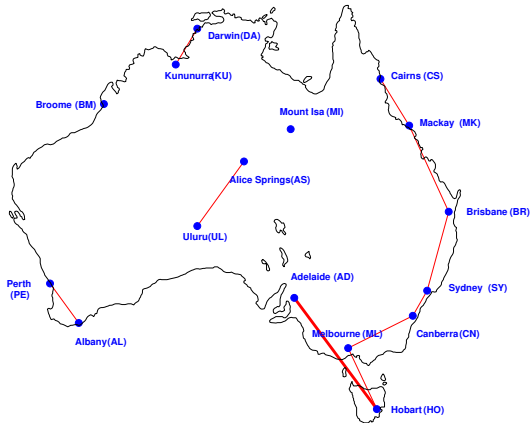
# The Cheapest-Link Algorithm



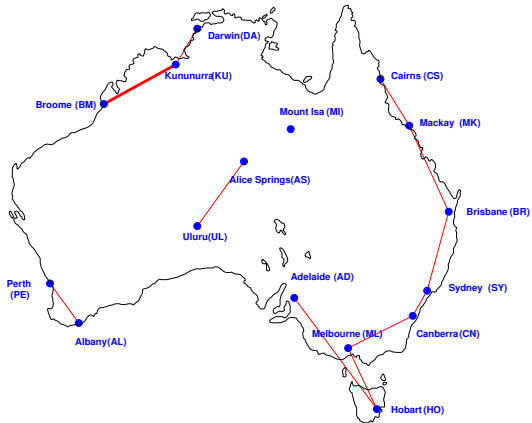
# The Cheapest-Link Algorithm



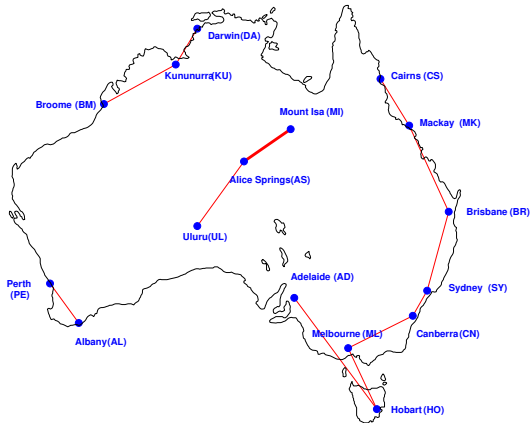
# The Cheapest-Link Algorithm



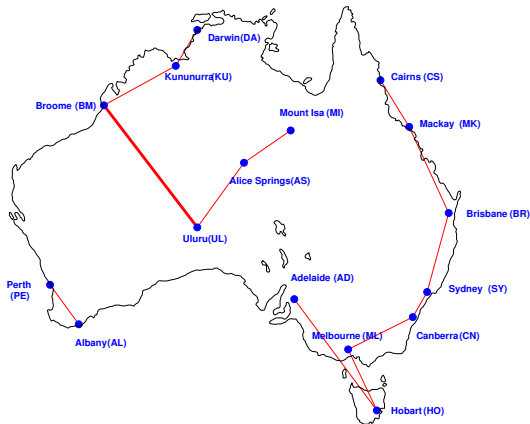
# The Cheapest-Link Algorithm



# The Cheapest-Link Algorithm

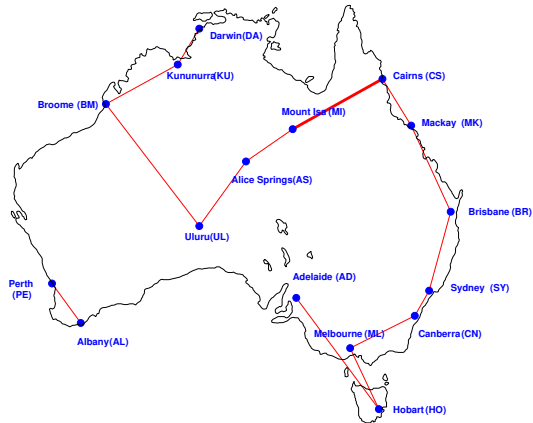


# The Cheapest-Link Algorithm

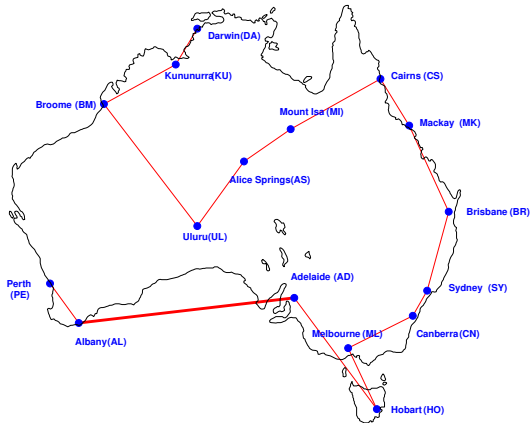


The edges we add get longer and longer.

# The Cheapest-Link Algorithm

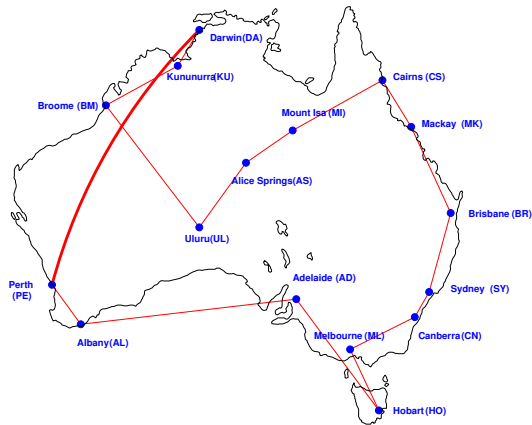


# The Cheapest-Link Algorithm



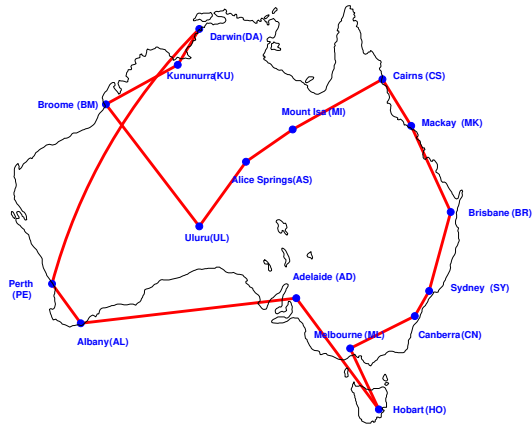


# The Cheapest-Link Algorithm



The last edge is very long indeed. . .

# The Cheapest-Link Algorithm



... but finally the circuit is complete.

# The Cheapest-Link Algorithm

Here is the same example, but with the links shown in a spreadsheet rather than pictorially.

(This way we don't have to worry about the figure not being to scale!)

At each stage, we look for the smallest number we haven't yet used (always being careful not to draw three edges to any vertex, or to close the circuit prematurely).



# The Cheapest-Link Algorithm

Step 1: Add the cheapest link: Sydney–Canberra

		Distance (km)																
		AD	AL	AS	BT	BM	CS	CN	DA	HO	KU	MK	ML	MI	PE	SY	UL	
3	Adelaide	AD	2673	1533	2045	2483	3352	1196	3022	1001	3219	2783	731	2742	2781	1412	1578	
4	Albany	AL	2673	3588	3588	4349	1943	5656	3846	4614	3674	3787	5087	3404	5106	409	3970	3633
5	Alice Springs	AS	1533	3588		3038	2483	2457	3706	1489	2534	1686	2505	2264	1209	3696	3830	443
6	Brisbane	BT	2045	4349	3038		3317	1716	1261	3463	1944	3660	976	1674	1829	4457	1001	3254
7	Broome	BM	2483	1943	2483	3317		2496	3275	1803	3636	1045	2840	3124	1834	2389	3373	1223
8	Cairns	CS	3352	5656	2457	1716	2496		2568	2882	3251	3079	740	2981	1248	5764	2495	2900
9	Canberra	CN	1196	3846	3706	1261	3275	2568		4195	918	4392	1999	648	2561	3954	2866	2751
10	Darwin	DA	3022	4614	1489	3463	1803	2882	4195		4023	827	2930	3753	1634	4205	4034	1932
11	Hobart	HO	1001	3674	2534	1944	3636	3251	918	4023		4220	2682	609	3075	3782	1142	2579
12	Kununurra	KU	3219	3787	1686	3660	1045	3079	4392	827	4220		3127	3950	1831	3378	4516	2129
13	Mackay	MK	2783	5087	2505	976	2840	740	1999	2930	2682	3127		2412	1296	5195	1926	2948
14	Melbourne	ML	731	3404	2264	1674	3124	2981	648	3753	609	3950	2412		2805	3512	872	2309
15	Mount Isa	MI	2742	5106	1209	1829	1834	1248	2561	1634	3075	1831	1296	2805		4905	2400	1652
16	Perth	PE	2781	409	3696	4457	2389	5764	3954	4205	3782	3378	5195	3512	4905		4078	3741
17	Sydney	SY	1412	3970	3830	1001	3373	2495	2866	4034	1142	4516	1926	872	2400	4078		2875
18	Uluru	UL	1578	3633	443	3254	1223	2900	2751	1932	2579	2129	2948	2309	1652	3741	2875	
19																		
20																		
21																		
		CN ↔ SY																

# The Cheapest-Link Algorithm

Step 2: Add the next cheapest link: Albany–Perth

Distance (km)

	AD	AL	AS	BT	BM	CS	CN	DA	HO	KU	MK	ML	MI	PE	SY	UL
Adelaide AD		2673	1533	2045	2483	3352	1196	3022	1001	3219	2783	731	2742	2781	1412	1578
Albany AL	2673		3588	4349	1943	5656	3846	4614	3674	3787	5087	3404	5106	409	3970	3633
Alice Springs AS	1533	3588		3038	2483	2457	3706	1489	2534	1686	2505	2264	1209	3696	3830	443
Brisbane BT	2045	4349	3038		3317	1716	1261	3463	1944	3660	976	1674	1829	4457	1001	3254
Broome BM	2483	1943	2483	3317		2496	3275	1803	3636	1045	2840	3124	1834	2389	3373	1223
Cairns CS	3352	5656	2457	1716	2496		2568	2882	3251	3079	740	2981	1248	5764	2495	2900
Canberra CN	1196	3846	3706	1261	3275	2568		4195	918	4392	1999	648	2561	3954	286	2751
Darwin DA	3022	4614	1489	3463	1803	2882	4195		4023	827	2930	3753	1634	4205	4034	1932
Hobart HO	1001	3674	2534	1944	3636	3251	918	4023		4220	2682	609	3075	3782	1142	2579
Kununurra KU	3219	3787	1686	3660	1045	3079	4392	827	4220		3127	3950	1831	3378	4516	2129
Mackay MK	2783	5087	2505	976	2840	740	1999	2930	2682	3127		2412	1296	5195	1926	2948
Melbourne ML	731	3404	2264	1674	3124	2981	648	3753	609	3950	2412		2805	3512	872	2309
Mount Isa MI	2742	5106	1209	1829	1834	1248	2561	1634	3075	1831	1296	2805		4905	2400	1652
Perth PE	2781	409	3696	4457	2389	5764	3954	4205	3782	3378	5195	3512	4905		4078	3741
Sydney SY	1412	3970	3830	1001	3373	2495	286	4034	1142	4516	1926	872	2400	4078		2875
Uluru UL	1578	3633	443	3254	1223	2900	2751	1932	2579	2129	2948	2309	1652	3741	2875	

CN ↔ SY    AL ↔ PE

# The Cheapest-Link Algorithm

Step 3: Add the next cheapest link: Alice Spings–Uluru

The screenshot shows a spreadsheet titled "cheapest-link.xls (Read-Only)". The data is organized as follows:

		Distance (km)																
		AD	AL	AS	BT	BM	CS	CN	DA	HO	KU	MK	ML	MI	PE	SY	UL	
3	Adelaide	AD	2673	1533	2045	2483	3352	1196	3022	1001	3219	2783	731	2742	2781	1412	1578	
4	Albany	AL	2673	3588	3588	4349	1943	5656	3846	4614	3674	3787	5087	3404	5106	409	3970	3633
5	Alice Spings	AS	1533	3588		3038	2483	2457	3706	1489	2534	1686	2505	2264	1209	3696	3830	443
6	Brisbane	BT	2045	4349	3038		3317	1716	1261	3463	1944	3660	976	1674	1829	4457	1001	3254
7	Broome	BM	2483	1943	2483	3317		2496	3275	1803	3636	1045	2840	3124	1834	2389	3373	1223
8	Cairns	CS	3352	5656	2457	1716	2496		2568	2882	3251	3079	740	2981	1248	5764	2495	2900
9	Canberra	CN	1196	3846	3706	1261	3275	2568		4195	918	4392	1999	648	2561	3954	286	2751
10	Darwin	DA	3022	4614	1489	3463	1803	2882	4195		4023	827	2930	3753	1634	4205	4034	1932
11	Hobart	HO	1001	3674	2534	1944	3636	3251	918	4023		4220	2682	609	3075	3782	1142	2579
12	Kununurra	KU	3219	3787	1686	3660	1045	3079	4392	827	4220		3127	3950	1831	3378	4516	2129
13	Mackay	MK	2783	5087	2505	976	2840	740	1999	2930	2682	3127		2412	1296	5195	1926	2948
14	Melbourne	ML	731	3404	2264	1674	3124	2981	648	3753	609	3950	2412		2805	3512	872	2309
15	Mount Isa	MI	2742	5106	1209	1829	1834	1248	2561	1634	3075	1831	1296	2805		4905	2400	1652
16	Perth	PE	2781	409	3696	4457	2389	5764	3954	4205	3782	3378	5195	3512	4905		4078	3741
17	Sydney	SY	1412	3970	3830	1001	3373	2495	286	4034	1142	4516	1926	872	2400	4078		2875
18	Uluru	UL	1578	3633	443	3254	1223	2900	2751	1932	2579	2129	2948	2309	1652	3741	2875	

At the bottom of the spreadsheet, there are three summary links: "CN ↔ SY", "AL ↔ PE", and "AS ↔ UL". The "AS ↔ UL" link is highlighted in green, corresponding to the highlighted cell in the matrix above.

# The Cheapest-Link Algorithm

Step 4: Add the next cheapest link: Hobart–Melbourne

cheapest-link.xls (Read-Only)

		Distance (km)																
		AD	AL	AS	BT	BM	CS	CN	DA	HO	KU	MK	ML	MI	PE	SY	UL	
3	Adelaide	AD	2673	1533	2045	2483	3352	1196	3022	1001	3219	2783	731	2742	2781	1412	1578	
4	Albany	AL	2673	3588	3588	4349	1943	5656	3846	4614	3674	3787	5087	3404	5106	409	3970	3633
5	Alice Springs	AS	1533	3588	3038	2483	2457	3706	1489	2534	1686	2505	2264	1209	3696	3830	443	
6	Brisbane	BT	2045	4349	3038	3317	1716	1261	3463	1944	3660	976	1674	1829	4457	1001	3254	
7	Broome	BM	2483	1943	2483	3317	2496	3275	1803	3636	1045	2840	3124	1834	2389	3373	1223	
8	Cairns	CS	3352	5656	2457	1716	2496	2568	2882	3251	3079	740	2981	1248	5764	2495	2900	
9	Canberra	CN	1196	3846	3706	1261	3275	2568	4195	918	4392	1999	648	2561	3954	286	2751	
10	Darwin	DA	3022	4614	1489	3463	1803	2882	4195	4023	827	2930	3753	1634	4205	4034	1932	
11	Hobart	HO	1001	3674	2534	1944	3636	3251	918	4023	4220	2682	609	3075	3782	1142	2579	
12	Kununurra	KU	3219	3787	1686	3660	1045	3079	4392	827	4220	3127	3950	1831	3378	4516	2129	
13	Mackay	MK	2783	5087	2505	976	2840	740	1999	2930	2682	3127	2412	1296	5195	1926	2948	
14	Melbourne	ML	731	3404	2264	1674	3124	2981	648	3753	609	3950	2412	2805	3512	872	2309	
15	Mount Isa	MI	2742	5106	1209	1829	1834	1248	2561	1634	3075	1831	1296	2805	4905	2400	1652	
16	Perth	PE	2781	409	3696	4457	2389	5764	3954	4205	3782	3378	5195	3512	4905	4078	3741	
17	Sydney	SY	1412	3970	3830	1001	3373	2495	286	4034	1142	4516	1926	872	2400	4078	2875	
18	Uluru	UL	1578	3633	443	3254	1223	2900	2751	1932	2579	2129	2948	2309	1652	2875		
19																		
20			CN ↔ SY	AL ↔ PE	AS ↔ UL				HO ↔ ML									

Distance Table | Step 1 | Step 2 | Step 3 | Step 4 | Step 5 | Step 10 | 51



# The Cheapest-Link Algorithm

Step 5: Add the cheapest link: Canberra-Melbourne

		Distance (km)																	
		AD	AL	AS	BT	BM	CS	CN	DA	HO	KU	MK	ML	MI	PE	SY	UL		
3	Adelaide	AD	2673	1533	2045	2483	3352	1196	3022	1001	3219	2783	731	2742	2781	1412	1578		
4	Albany	AL	2673	3588	3588	4349	1943	5656	3846	4614	3674	3787	5087	3404	5106	409	3970	3633	
5	Alice Springs	AS	1533	3588		3038	2483	2457	3706	1489	2534	1686	2505	2264	1209	3696	3830	443	
6	Brisbane	BT	2045	4349	3038		3317	1716	1261	3463	1944	3660	976	1674	1829	4457	1001	3254	
7	Broome	BM	2483	1943	2483	3317		2496	3275	1803	3636	1045	2840	3124	1834	2389	3373	1223	
8	Cairns	CS	3352	5656	2457	1716	2496		2568	2882	3251	3079	740	2981	1248	5764	2495	2900	
9	Canberra	CN	1196	3846	3706	1261	3275	2568		4195	918	4392	1999	648	2561	3954	286	2751	
10	Darwin	DA	3022	4614	1489	3463	1803	2882	4195		4023	827	2930	3753	1634	4205	4034	1932	
11	Hobart	HO	1001	3674	2534	1944	3636	3251	918	4023		4220	2682	609	3075	3782	1142	2579	
12	Kununurra	KU	3219	3787	1686	3660	1045	3079	4392	827	4220		3127	3950	1831	3378	4516	2129	
13	Mackay	MK	2783	5087	2505	976	2840	740	1999	2930	2682	3127		2412	1296	5195	1926	2948	
14	Melbourne	ML	731	3404	2264	1674	3124	2981	648	3753	609	3950	2412		2805	3512	872	2309	
15	Mount Isa	MI	2742	5106	1209	1829	1834	1248	2561	1634	3075	1831	1296	2805		4905	2400	1652	
16	Perth	PE	2781	409	3696	4457	2389	5764	3954	4205	3782	3378	5195	3512	4905		4078	3741	
17	Sydney	SY	1412	3970	3830	1001	3373	2495	286	4034	1142	4516	1926	872	2400	4078		2875	
18	Uluru	UL	1578	3633	443	3254	1223	2900	2751	1932	2579	2129	2948	2309	1652	3741	2875		
19																			
20																			
21																			
22																			

SY ↔ CN ↔ ML ↔ HO      AL ↔ PE      AS ↔ UL

# The Cheapest-Link Algorithm

CN and ML both have two edges, so we are done with them

		Distance (km)																
		AD	AL	AS	BT	BM	CS	CN	DA	HO	KU	MK	ML	MI	PE	SY	UL	
3	Adelaide	AD	2673	1533	2045	2483	3352	1196	3022	1001	3219	2783	731	2742	2781	1412	1578	
4	Albany	AL	2673	3588	3588	4349	1943	5656	3846	4614	3674	3787	5087	3404	5106	409	3970	3633
5	Alice Springs	AS	1533	3588		3038	2483	2457	3706	1489	2534	1686	2505	2264	1209	3696	3830	443
6	Brisbane	BT	2045	4349	3038		3317	1716	1261	3463	1944	3660	976	1674	1829	4457	1001	3254
7	Broome	BM	2483	1943	2483	3317		2496	3275	1803	3636	1045	2840	3124	1834	2389	3373	1223
8	Cairns	CS	3352	5656	2457	1716	2496		2568	2882	3251	3079	740	2981	1248	5764	2495	2900
9	Canberra	CN	1196	3846	3706	1261	3275	2568		4195	918	4392	1999	648	2561	3954	286	2751
10	Darwin	DA	3022	4614	1489	3463	1803	2882	4195		4023	827	2930	3753	1634	4205	4034	1932
11	Hobart	HO	1001	3674	2534	1944	3636	3251	918	4023		4220	2682	609	3075	3782	1142	2579
12	Kununurra	KU	3219	3787	1686	3660	1045	3079	4392	827	4220		3127	3950	1831	3378	4516	2129
13	Mackay	MK	2783	5087	2505	976	2840	740	1999	2930	2682	3127		2412	1296	5195	1926	2948
14	Melbourne	ML	731	3404	2264	1674	3124	2981	648	3753	609	3950	2412		2805	3512	872	2309
15	Mount Isa	MI	2742	5106	1209	1829	1834	1248	2561	1634	3075	1831	1296	2805		4905	2400	1652
16	Perth	PE	2781	409	3696	4457	2389	5764	3954	4205	3782	3378	5195	3512	4905		4078	3741
17	Sydney	SY	1412	3970	3830	1001	3373	2495	286	4034	1142	4516	1926	872	2400	4078		2875
18	Uluru	UL	1578	3633	443	3254	1223	2900	2751	1932	2579	2129	2948	2309	1652	3741	2875	
19																		
20																		
21																		
		SY ↔ CN ↔ ML ↔ HO				AL ↔ PE				AS ↔ UL								

# The Cheapest-Link Algorithm

Keep adding links until the circuit is complete

cheapest-link.xls (Read-Only)

		Distance (km)																
		AD	AL	AS	BT	BM	CS	CN	DA	HO	KU	MK	ML	MI	PE	SY	UL	
3	Adelaide	AD	2673	1533	2045	2483	3352	1196	3022	1001	3219	2783	731	2742	2781	1412	1578	
4	Albany	AL	2673	3588	3588	4349	1943	5656	3846	4614	3674	5087	3404	5106	409	3970	3633	
5	Alice Springs	AS	1533	3588	3038	2483	2457	3706	1489	2534	1686	2505	2264	1209	3696	3830	443	
6	Brisbane	BT	2045	4349	3038	3317	1716	1261	3463	1944	3660	976	1674	1829	4457	1001	3254	
7	Broome	BM	2483	1943	2483	3317	2496	3275	1803	3636	1045	2840	3124	1834	2389	3373	1223	
8	Cairns	CS	3352	5656	2457	1716	2496	2568	2882	3251	3079	740	2981	1248	5764	2495	2900	
9	Canberra	CN	1196	3846	3706	1261	3275	2568	4195	918	4392	1999	648	2561	3954	286	2751	
10	Darwin	DA	3022	4614	1489	3463	1803	2882	4195	4023	827	2930	3753	1634	4205	4034	1932	
11	Hobart	HO	1001	3674	2534	1944	3636	3251	918	4023	4220	2682	609	3075	3782	1142	2579	
12	Kununurra	KU	3219	3787	1686	3660	1045	3079	4392	827	4220	3127	3950	1831	3378	4516	2129	
13	Mackay	MK	2783	5087	2505	976	2840	740	1999	2930	2682	3127	2412	1296	5195	1926	2948	
14	Melbourne	ML	731	3404	2264	1674	3124	2981	648	3753	609	3950	2412	2805	3512	872	2309	
15	Mount Isa	MI	2742	5106	1209	1829	1834	1248	2561	1634	3075	1831	1296	2805	4905	2400	1652	
16	Perth	PE	2781	409	3696	4457	2389	5764	3954	4205	3782	3378	5195	3512	4905	4078	3741	
17	Sydney	SY	1412	3970	3830	1001	3373	2495	286	4034	1142	4516	1926	872	2400	4078	2875	
18	Uluru	UL	1578	3633	443	3254	1223	2900	2751	1932	2579	2129	2948	2309	1652	3741	2875	
20			SY ↔ CN ↔ ML ↔ HO				AL ↔ PE				AS ↔ UL				CS ↔ MK			

Summary: Step 1 Step 2 Step 3 Step 4 Step 5 Step 6 Step 7 5

Normal View Ready

# The Cheapest-Link Algorithm

Keep adding links until the circuit is complete

		Distance (km)																
		AD	AL	AS	BT	BM	CS	CN	DA	HO	KU	MK	ML	MI	PE	SY	UL	
3	Adelaide	AD	2673	1533	2045	2483	3352	1196	3022	1001	3219	2783	731	2742	2781	1412	1578	
4	Albany	AL	2673	3588	3588	4349	1943	5656	3846	4614	3674	5087	3404	5106	409	3970	3633	
5	Alice Springs	AS	1533	3588	3038	2483	2457	3706	1489	2534	1686	2505	2264	1209	3696	3830	443	
6	Brisbane	BT	2045	4349	3038	3317	1716	1261	3463	1944	3660	976	1674	1829	4457	1001	3254	
7	Broome	BM	2483	1943	2483	3317	2496	3275	1803	3636	1045	2840	3124	1834	2389	3373	1223	
8	Cairns	CS	3352	5656	2457	1716	2496	2568	2882	3251	3079	740	2981	1248	5764	2495	2900	
9	Canberra	CN	1196	3846	3706	1261	3275	2568	4195	918	4392	1999	648	2561	3954	286	2761	
10	Darwin	DA	3022	4614	1489	3463	1803	2882	4195	4023	827	2930	3753	1634	4205	4034	1932	
11	Hobart	HO	1001	3674	2534	1944	3636	3251	918	4023	4220	2682	609	3075	3782	1142	2579	
12	Kununurra	KU	3219	3787	1686	3660	1045	3079	4392	827	4220	3127	3950	1831	3378	4516	2129	
13	Mackay	MK	2783	5087	2505	976	2840	740	1999	2930	2682	3127	2412	1296	5195	1926	2948	
14	Melbourne	ML	731	3404	2264	1674	3124	2981	648	3753	609	3950	2412	2905	3512	872	2309	
15	Mount Isa	MI	2742	5106	1209	1829	1834	1248	2561	1634	3075	1831	1296	2805	4905	2400	1652	
16	Perth	PE	2781	409	3696	4457	2389	5764	3954	4205	3782	3378	5195	3512	4905	4078	3741	
17	Sydney	SY	1412	3970	3830	1001	3373	2495	286	4034	1142	4516	1926	872	2400	4078	2875	
18	Uluru	UL	1578	3633	443	3254	1223	2900	2761	1932	2579	2129	2948	2309	1652	3741	2875	
19																		
20			SY ↔ CN ↔ ML ↔ HO				AL ↔ PE			AS ↔ UL			CS ↔ MK			DA ↔ KU		

# The Cheapest-Link Algorithm

Keep adding links until the circuit is complete

	AD	AL	AS	BT	BM	CS	CN	DA	HO	KU	MK	ML	MI	PE	SY	UL	
Adelaide	AD	2673	1533	2045	2483	3352	1196	3022	1001	3219	2783	731	2742	2781	1412	1578	
Albany	AL	2673	3588	4349	1943	5656	3846	4614	3674	3787	5087	3404	5106	409	3970	3633	
Alice Springs	AS	1533	3588	3038	2483	2457	3706	1489	2534	1686	2505	2264	1209	3696	3830	443	
Brisbane	BT	2045	4349	3038		3317	1716	1261	3463	1944	3660	976	1674	1829	4457	1001	3254
Broome	BM	2483	1943	2483	3317		2496	3275	1803	3636	1045	2840	3124	1834	2389	3373	1223
Cairns	CS	3352	5656	2457	1716	2496		2568	2882	3251	3079	740	2981	1248	5764	2495	2900
Canberra	CN	1196	3846	3706	1261	3275	2568		4195	918	4392	1999	648	2561	3954	286	2751
Darwin	DA	3022	4614	1489	3463	1803	2882	4195		4023	827	2930	3753	1634	4205	4034	1932
Hobart	HO	1001	3674	2534	1944	3636	3251	918	4023		4220	2682	609	3075	3782	1142	2579
Kununurra	KU	3219	3787	1686	3660	1045	3079	4392	827	4220		3127	3950	1831	3378	4516	2129
Mackay	MK	2783	5087	2505	976	2840	740	1999	2930	2682	3127		2412	1296	5195	1926	2948
Melbourne	ML	731	3404	2264	1674	3124	2981	648	3753	609	3950	2412		2805	3512	872	2309
Mount Isa	MI	2742	5106	1209	1829	1834	1248	2561	1634	3075	1831	1296	2805		4905	2400	1652
Perth	PE	2781	409	3696	4457	2389	5764	3954	4205	3782	3378	5195	3512	4905		4078	3741
Sydney	SY	1412	3970	3830	1001	3373	2495	286	4034	1142	4516	1926	872	2400	4078		2875
Uluru	UL	1578	3633	443	3254	1223	2900	2751	1932	2579	2129	2948	2309	1652	3741	2875	

SY ↔ CN ↔ ML ↔ HO      AL ↔ PE      AS ↔ UL      CS ↔ MK ↔ BT      DA ↔ KU

# The Cheapest-Link Algorithm

Keep adding links until the circuit is complete

		Distance (km)																	
		AD	AL	AS	BT	BM	CS	CN	DA	HO	KU	MK	ML	MI	PE	SY	UL		
3	Adelaide	AD	2673	1533	2045	2483	3352	1196	3022	1001	3219	2783	731	2742	2781	1412	1578		
4	Albany	AL	2673	3588	3588	4349	1943	5656	3846	4614	3674	3787	5087	3404	5106	409	3970	3633	
5	Alice Springs	AS	1533	3588		3038	2483	2457	3706	1489	2534	1686	2505	2264	1209	3696	3830	443	
6	Brisbane	BT	2045	4349	3038		3317	1716	1261	3463	1944	3660	976	1674	1829	4457	1001	3254	
7	Broome	BM	2483	1943	2483	3317		2496	3275	1803	3636	1045	2840	3124	1834	2389	3373	1223	
8	Cairns	CS	3352	5656	2457	1716	2496		2568	2882	3251	3079	740	2981	1248	5764	2495	2900	
9	Canberra	CN	1196	3846	3706	1261	3275	2568		4195	918	4392	1999	648	2561	3954	286	2751	
10	Darwin	DA	3022	4614	1489	3463	1803	2882	4195		4023	827	2930	3753	1634	4205	4034	1932	
11	Hobart	HO	1001	3674	2534	1944	3636	3251	918	4023		4220	2682	609	3075	3782	1142	2579	
12	Kununurra	KU	3219	3787	1686	3660	1045	3079	4392	827	4220		3127	3950	1831	3378	4516	2129	
13	Mackay	MK	2783	5087	2505	976	2840	740	1999	2930	2682	3127		2412	1296	5195	1926	2948	
14	Melbourne	ML	731	3404	2264	1674	3124	2981	648	3753	609	3950	2412		2805	3512	872	2309	
15	Mount Isa	MI	2742	5106	1209	1829	1834	1248	2561	1634	3075	1831	1296	2805		4905	2400	1652	
16	Perth	PE	2781	409	3696	4457	2389	5764	3954	4205	3782	3378	5195	3512	4905		4078	3741	
17	Sydney	SY	1412	3970	3830	1001	3373	2495	286	4034	1142	4516	1926	872	2400	4078		2875	
18	Uluru	UL	1578	3633	443	3254	1223	2900	2751	1932	2579	2129	2948	2309	1652	3741	2875		
20			SY ↔ CN ↔ ML ↔ HO ↔ AD				AL ↔ PE			AS ↔ UL			CS ↔ MK ↔ BT			DA ↔ KU			

# The Cheapest-Link Algorithm

Two more links to go!

cheapest-link.xls (Read-Only)

		Distance (km)																
		AD	AL	AS	BT	BM	CS	CN	DA	HO	KU	MK	ML	MI	PE	SY	UL	
3	Adelaide	AD	2673	1533	2045	2483	3352	1196	3022	1001	3219	2783	731	2742	2781	1412	1578	
4	Albany	AL	2673	3588	4349	1943	5656	3846	4614	3674	3787	5087	3404	5106	409	3970	3633	
5	Alice Springs	AS	1533	3588	3038	3317	2457	3706	1489	2534	1686	2505	2264	1209	3696	3830	443	
6	Brisbane	BT	2045	4349	3038	3317	1716	1261	3463	1944	3660	976	1674	1829	4457	1001	3254	
7	Broome	BM	2483	1943	2483	3317	2496	3275	1803	3636	1045	2840	3124	1834	2389	3373	1223	
8	Cairns	CS	3352	5656	2457	1716	2496	2568	2882	3251	3079	740	2981	1248	5764	2495	2900	
9	Canberra	CN	1196	3846	3706	1261	3275	2568	4195	918	4392	1999	648	2561	3954	286	2751	
10	Darwin	DA	3022	4614	1489	3463	1803	2882	4195	4023	827	2930	3753	1634	4205	4034	1932	
11	Hobart	HO	1001	3674	2534	1944	3636	3251	918	4023		4220	2682	609	3075	3782	1142	2579
12	Kununurra	KU	3219	3787	1686	3660	1045	3079	4392	827	4220		3127	3950	1831	3378	4516	2129
13	Mackay	MK	2783	5087	2505	976	2840	740	1999	2930	2682	3127	2412	1296	5195	1926	2948	
14	Melbourne	ML	731	3404	2264	1674	3124	2981	648	3753	609	3950	2412		2805	3512	872	2309
15	Mount Isa	MI	2742	5106	1209	1829	1834	1248	2561	1634	3075	1831	1296	2805		4905	2400	1652
16	Perth	PE	2781	409	3696	4457	2389	5764	3954	4205	3782	3378	5195	3512	4905		4078	3741
17	Sydney	SY	1412	3970	3830	1001	3373	2495	286	4034	1142	4516	1926	872	2400	4078		2875
18	Uluru	UL	1578	3633	443	3254	1223	2900	2751	1932	2579	2129	2948	2309	1652	3741	2875	
20	DA ↔ KU ↔ BM ↔ UL ↔ AS ↔ MI ↔ CS ↔ MK ↔ BT ↔ SY ↔ CN ↔ ML ↔ HO ↔ AD															AL ↔ PE		

Step 4 Step 5 Step 6 Step 7 Step 8 Step 9 Step 14

# The Cheapest-Link Algorithm

Next-to-last step: add AD-AL

		Distance (km)																			
		AD	AL	AS	BT	BM	CS	CN	DA	HO	KU	MK	ML	MI	PE	SY	UL				
3	Adelaide	AD	2673	1533	2045	2483	3352	1196	3022	1001	3219	2783	731	2742	2781	1412	1578				
4	Albany	AL	2673	3588	4349	1943	5656	3846	4614	3674	3787	5087	3404	5106	409	3696	3830	3633			
5	Alice Springs	AS	1533	3588	3038	2483	2457	3706	1489	2534	1686	2505	2264	1209	3696	3830	443				
6	Brisbane	BT	2045	4349	3038	3317	1716	1261	3463	1944	3660	976	1674	1829	4457	1001	3254				
7	Broome	BM	2483	1943	2483	3317	2496	3275	1803	3636	1045	2840	3124	1834	2389	3373	1223				
8	Cairns	CS	3352	5656	2457	1716	2496	2568	2882	3251	3079	740	2981	1248	5764	2495	2900				
9	Canberra	CN	1196	3846	3706	1261	3275	2568	4195	918	4392	1999	648	2561	3954	286	2751				
10	Darwin	DA	3022	4614	1489	3463	1803	2882	4195	4023	827	2930	3753	1634	4205	4034	1932				
11	Hobart	HO	1001	3674	2534	1944	3636	3251	918	4023	4220	2682	609	3075	3782	1142	2579				
12	Kununurra	KU	3219	3787	1686	3660	1045	3079	4392	827	4220	3127	3950	1831	3378	4516	2129				
13	Mackay	MK	2783	5087	2505	976	2840	740	1999	2930	2682	3127	2412	1296	5195	1926	2948				
14	Melbourne	ML	731	3404	2264	1674	3124	2981	648	3753	609	3950	2412	2805	3512	872	2309				
15	Mount Isa	MI	2742	5106	1209	1829	1834	1248	2561	1634	3075	1831	1296	2805	4905	2400	1652				
16	Perth	PE	2781	409	3696	4457	2389	5764	3954	4205	3782	3378	5195	3512	4905	4078	3741				
17	Sydney	SY	1412	3970	3830	1001	3373	2495	286	4034	1142	4516	1926	872	2400	4078	2875				
18	Uluru	UL	1578	3633	443	3254	1223	2900	2751	1932	2579	2129	2948	2309	1652	3741	2875				
20																					
21																					

DA ↔ KU ↔ BM ↔ UL ↔ AS ↔ MI ↔ CS ↔ MK ↔ BT ↔ SY ↔ CN ↔ ML ↔ HO ↔ AD ↔ AL ↔ PE



# The Cheapest-Link Algorithm

Last step: close the circuit

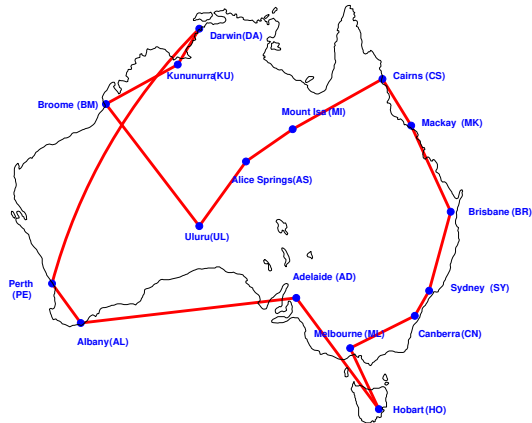
The screenshot shows a spreadsheet titled "cheapest-link.xls (Read-Only)". The main data is a distance matrix for 18 cities: Adelaide (AD), Albany (AL), Alice Springs (AS), Brisbane (BT), Broome (BM), Cairns (CS), Canberra (CN), Darwin (DA), Hobart (HO), Kununurra (KU), Mackay (MK), Melbourne (ML), Mount Isa (MI), Perth (PE), Sydney (SY), and Uluru (UL). The distances are in kilometers. The final step of the Cheapest-Link Algorithm is shown as a circuit path: DA → KU → BM → UL → AS → MI → CS → MK → BT → SY → CN → ML → HO → AD → AL → PE → DA.

	AD	AL	AS	BT	BM	CS	CN	DA	HO	KU	MK	ML	MI	PE	SY	UL
Adelaide (AD)		2673	1533	2045	2483	3352	1196	3022	1001	3219	2783	731	2742	2781	1412	1578
Albany (AL)	2673		3588	4349	1943	5656	3846	4614	3674	3787	5087	3404	5106	409	3970	3633
Alice Springs (AS)	1533	3588		3038	2483	2457	3706	1489	2534	1686	2505	2264	1209	3696	3830	443
Brisbane (BT)	2045	4349	3038		3317	1716	1261	3463	1944	3660	976	1674	1829	4457	1001	3254
Broome (BM)	2483	1943	2483	3317		2496	3275	1803	3636	1045	2840	3124	1834	2389	3373	1223
Cairns (CS)	3352	5656	2457	1716	2496		2568	2882	3251	3079	740	2981	1248	5764	2495	2900
Canberra (CN)	1196	3846	3706	1261	3275	2568		4195	918	4392	1999	648	2561	3954	286	2751
Darwin (DA)	3022	4614	1489	3463	1803	2882	4195		4023	827	2930	3753	1634	4205	4034	1932
Hobart (HO)	1001	3674	2534	1944	3636	3251	918	4023		4220	2682	609	3075	3782	1142	2579
Kununurra (KU)	3219	3787	1686	3660	1045	3079	4392	827	4220		3127	3950	1831	3378	4516	2129
Mackay (MK)	2783	5087	2505	976	2840	740	1999	2930	2682	3127		2412	1296	5195	1926	2948
Melbourne (ML)	731	3404	2264	1674	3124	2981	648	3753	609	3950	2412		2805	3512	872	2309
Mount Isa (MI)	2742	5106	1209	1829	1834	1248	2561	1634	3075	1831	1296	2805		4905	2400	1652
Perth (PE)	2781	409	3696	4457	2389	5764	3954	4205	3782	3378	5195	3512	4905		4078	3741
Sydney (SY)	1412	3970	3830	1001	3373	2495	286	4034	1142	4516	1926	872	2400	4078		2875
Uluru (UL)	1578	3633	443	3254	1223	2900	2751	1932	2579	2129	2948	2309	1652	3741	2875	

DA → KU → BM → UL → AS → MI → CS → MK → BT → SY → CN → ML → HO → AD → AL → PE → DA

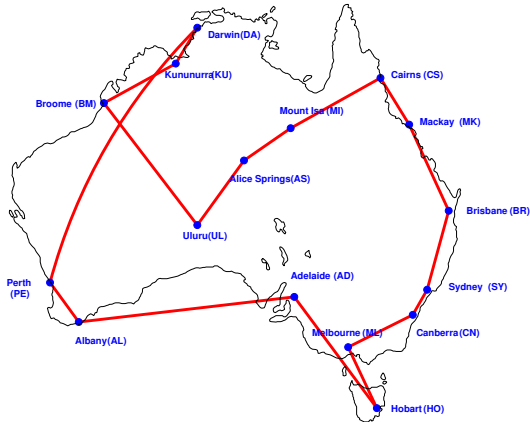
# The Cheapest-Link Algorithm

Here is the output of the Cheapest-Link Algorithm.



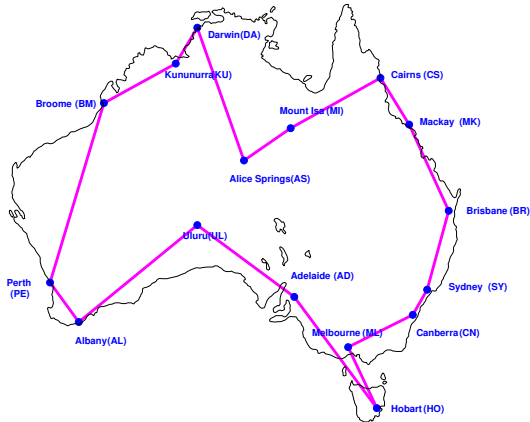
# The Cheapest-Link Algorithm

It is not optimal since it involves crossings...



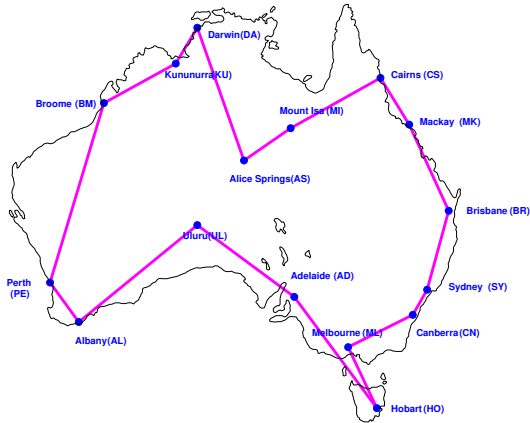
# The Cheapest-Link Algorithm

... this route is an improvement.



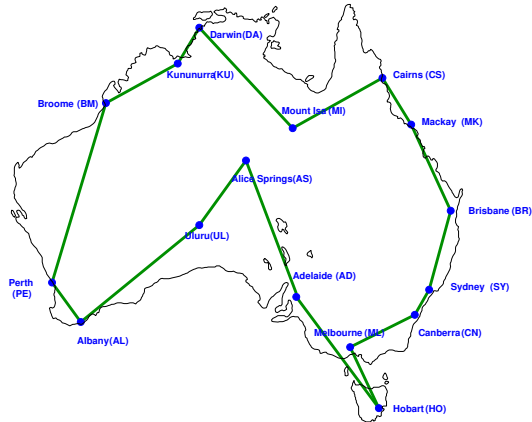
# The Cheapest-Link Algorithm

But it is unclear whether the new route is optimal.



# The Cheapest-Link Algorithm

For example, maybe this route is even better.



# Comparing Algorithms

Randomly chosen Hamilton circuit:	40,680 km
Hamilton circuit using NNA/Sydney:	21,049 km
Hamilton circuit using RNNA:	<b>18,459 km</b>
Hamilton circuit using CLA:	18,543 km

So CLA did not find an optimal circuit. But it is a reasonable method that might perform better in a different example.

**Might there be an even better Hamilton circuit? Can we find it without having to use brute force?**

# The Bad News

There is no known algorithm to solve the TSP that is both **optimal** and **efficient**.

- ▶ Brute-force is optimal but not efficient.
- ▶ NNA, RNNA, and CLA are efficient but not optimal.
- ▶ Maybe no optimal, efficient algorithm exists. . . or maybe it's out there but no one has found it yet. We don't know!