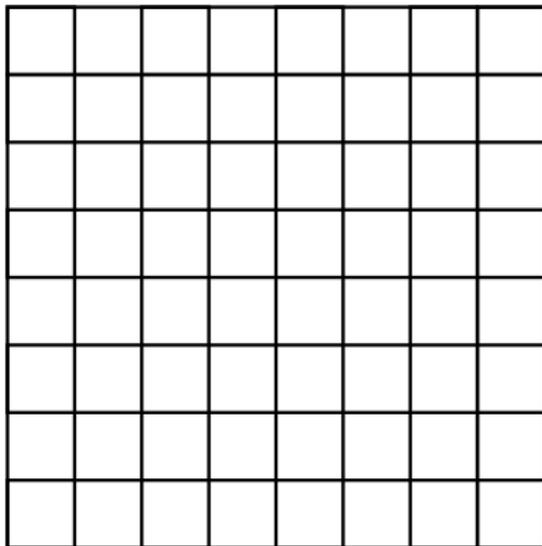
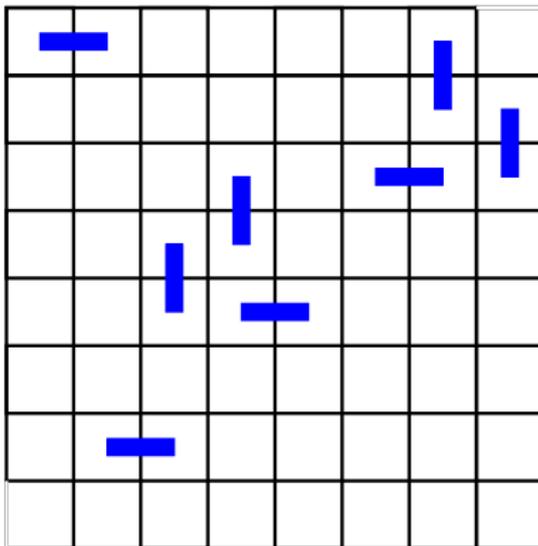


The Domino Problem

Take an 8×8 grid (64 squares).



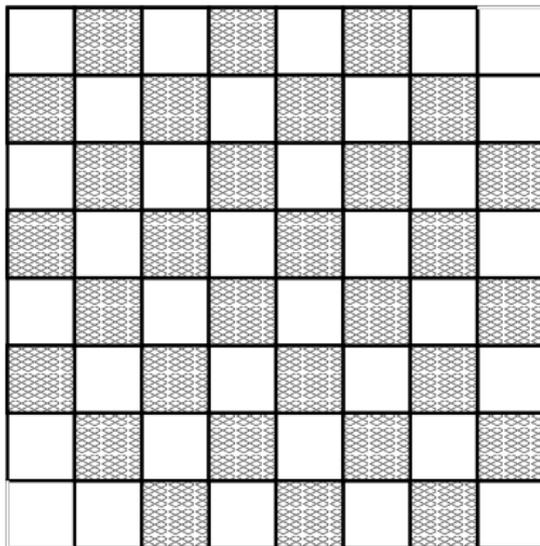
Is it possible to place 31 dominoes on the grid, with every domino covering two squares, so that every square gets covered exactly once?



The Domino Problem

The answer: **No.**

To see why not, color the grid like a chessboard.



The Domino Problem

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The Domino Problem

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- ▶ The two deleted squares were **both black**.
- ▶ So the new grid has 32 white squares, but only 30 black squares.
- ▶ Meanwhile, every domino has to cover one white and one black square.
- ▶ Therefore, it is impossible to cover the board with 31 dominoes!