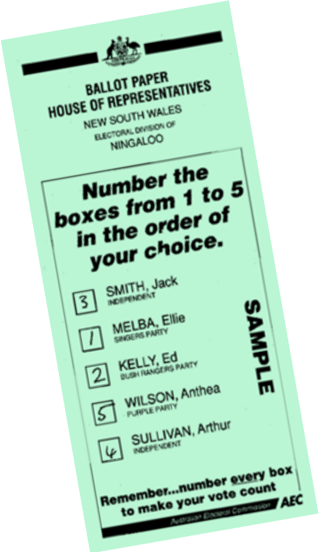
Part 1: The Mathematics of Social Choice

Voting and Elections



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- How can we use mathematics to design, analyze and compare different election methods?
- How can we use mathematics to say what "fair" means?
- Mathematical fact: No voting method can succeed in being completely fair all the time.

Example: 1998 Minnesota Gubernatorial Election

Candidate	Percentage of Votes[1]	
Jesse Ventura (I)	36.99%	(winner)
Norm Coleman (R)	34.29%	
Skip Humphrey (D)	28.09%	
All others	0.63%	

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- Given that the winner only received $\approx$ 37% of the votes, how sure can we be that the system produced an outcome that reflected the will of the voters?
- **Important Point**: As mathematicians, we are studying *election methods*. Who the particular *candidates* are, or which parties they belong to, doesn't matter.

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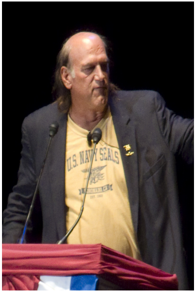
Who Is Society's Choice?



Ventura: 37%

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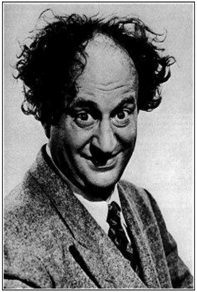

Larry: 37%

Curly: 34%

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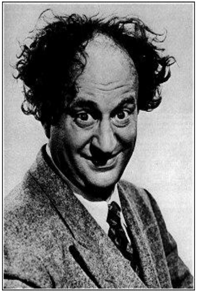

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Scenario 1: Supporters of both Curly and Moe would have listed Larry as their second choice. *Maybe Larry should be the winner.*

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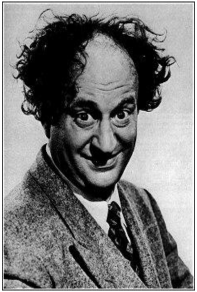


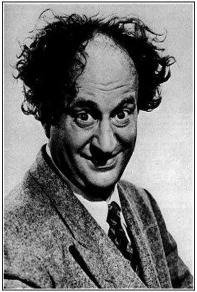
Larry: 37%

Curly: 34%

Moe: 28%

Scenario 2: Supporters of both Curly and Moe would have listed Larry as their last choice. *Maybe there should be a runoff between Curly and Moe.*

Who Is Society's Choice?



Larry: 37% **Curly: 34%** **Moe: 28%**

Scenario 3: One of the "others" would have been perfectly satisfactory to supporters of all three named candidates. *Maybe that candidate should be the winner.*

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Suppose that voters were allowed to rank all the candidates instead of having to choose just one.

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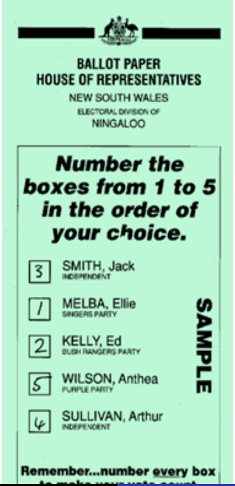

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- In what different ways might we use that additional information to design a voting method?
- How can we use mathematics to analyze whether a voting method is fair, or to compare methods to each other?
- How can we use mathematics to analyze how resistant a voting method is to **strategic voting**?

Strategic Voting

- In many situations, voters can affect election results by **strategic voting** (a.k.a. **insincere voting**).

  ("If my favorite candidate has no chance to win, then I will vote instead for someone I like less, but who has a chance to win.")

Example: The 2000 US Presidential Election

Candidate	Popular Vote (US)[2]	Popular Vote (FL)[3]	Electoral Votes
George W. Bush	47.87%	48.847%	271
Al Gore	48.38%	48.838%	266
Ralph Nader	2.74%	1.635%	0
All others	1.01%	0.680%	0

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"Most Nader supporters probably preferred Gore to Bush. If they had voted for Gore, then Gore might have won Florida."

"Some Nader supporters probably *did* vote for Gore. If they had voted sincerely, Bush might have won Florida easily."

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- Mathematics takes no stance on whether strategic voting is moral or immoral.

- The real problem with strategic voting is that it **reduces the effectiveness of the voting system itself**.

Impossibility Theorems

Gibbard-Satterthwaite Theorem: No voting method is completely resistant to strategic voting.

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- These "impossibility theorems" are what makes voting theory interesting. There is room for debate about which system is "best", but that debate should be informed by objective truth (that is, mathematics).

Ballots and Preference Schedules (§1.1)

Example (Tannenbaum, p.4): Electing a Math Club president.

- Candidates: Alisha (A), Boris (B), Carmen (C), Dave (D).
- Each of the 37 club members submits a ballot listing his or her first, second, third and fourth choices.
- Who should be the winner?

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st *A*	1st *B*	1st *A*	1st *C*	1st *B*	1st *C*	1st *A*	1st *B*	1st *C*	1st *A*	1st *C*	1st *D*	1st *A*	1st *A*	1st *C*	1st *A*	1st *C*	1st *D*
2nd *B*	2nd *D*	2nd *B*	2nd *B*	2nd *D*	2nd *B*	2nd *B*	2nd *D*	2nd *B*	2nd *B*	2nd *B*	2nd *C*	2nd *B*	2nd *B*	2nd *B*	2nd *B*	2nd *B*	2nd *C*
3rd *C*	3rd *C*	3rd *C*	3rd *D*	3rd *C*	3rd *D*	3rd *C*	3rd *C*	3rd *D*	3rd *C*	3rd *D*	3rd *B*	3rd *C*	3rd *C*	3rd *D*	3rd *C*	3rd *D*	3rd *B*
4th *D*	4th *A*	4th *D*	4th *A*	4th *A*	4th *A*	4th *D*	4th *A*	4th *A*	4th *D*	4th *A*	4th *A*	4th *D*	4th *D*	4th *A*	4th *D*	4th *A*	4th *A*

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st *C*	1st *A*	1st *D*	1st *D*	1st *C*	1st *C*	1st *D*	1st *A*	1st *D*	1st *C*	1st *A*	1st *D*	1st *B*	1st *A*	1st *C*	1st *A*	1st *A*	1st *D*	1st *A*
2nd *B*	2nd *B*	2nd *C*	2nd *C*	2nd *B*	2nd *B*	2nd *C*	2nd *B*	2nd *C*	2nd *B*	2nd *B*	2nd *C*	2nd *D*	2nd *B*	2nd *D*	2nd *B*	2nd *B*	2nd *C*	2nd *B*
3rd *D*	3rd *C*	3rd *B*	3rd *B*	3rd *D*	3rd *D*	3rd *B*	3rd *C*	3rd *B*	3rd *D*	3rd *C*	3rd *B*	3rd *C*	3rd *C*	3rd *B*	3rd *C*	3rd *C*	3rd *B*	3rd *C*
4th *A*	4th *D*	4th *A*	4th *A*	4th *A*	4th *A*	4th *A*	4th *D*	4th *A*	4th *A*	4th *D*	4th *A*	4th *A*	4th *D*	4th *A*	4th *D*	4th *D*	4th *A*	4th *D*

Voting Theory Terminology

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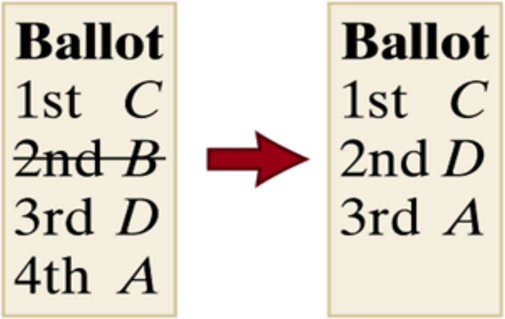
The Transitivity Assumption

Ballot	
1st	C
2nd	B
3rd	D
4th	A

The Elimination Assumption



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Preference schedule: A table of how many times each possible ballot was submitted.

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st *A*	1st *B*	1st *A*	1st *C*	1st *B*	1st *C*	1st *A*	1st *B*	1st *C*	1st *A*	1st *C*	1st *D*	1st *A*	1st *A*	1st *C*	1st *A*	1st *C*	1st *D*
2nd *B*	2nd *D*	2nd *B*	2nd *B*	2nd *D*	2nd *B*	2nd *B*	2nd *D*	2nd *B*	2nd *B*	2nd *B*	2nd *C*	2nd *B*	2nd *B*	2nd *B*	2nd *B*	2nd *B*	2nd *C*
3rd *C*	3rd *C*	3rd *C*	3rd *D*	3rd *C*	3rd *D*	3rd *C*	3rd *C*	3rd *D*	3rd *C*	3rd *D*	3rd *B*	3rd *C*	3rd *C*	3rd *D*	3rd *C*	3rd *D*	3rd *B*
4th *D*	4th *A*	4th *D*	4th *A*	4th *A*	4th *A*	4th *D*	4th *A*	4th *A*	4th *D*	4th *A*	4th *A*	4th *D*	4th *D*	4th *A*	4th *D*	4th *A*	4th *A*

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st *C*	1st *A*	1st *D*	1st *D*	1st *C*	1st *C*	1st *D*	1st *A*	1st *D*	1st *C*	1st *A*	1st *D*	1st *B*	1st *A*	1st *C*	1st *A*	1st *A*	1st *D*	1st *A*
2nd *B*	2nd *B*	2nd *C*	2nd *C*	2nd *B*	2nd *B*	2nd *C*	2nd *B*	2nd *C*	2nd *B*	2nd *B*	2nd *C*	2nd *D*	2nd *B*	2nd *D*	2nd *B*	2nd *B*	2nd *C*	2nd *B*
3rd *D*	3rd *C*	3rd *B*	3rd *B*	3rd *D*	3rd *D*	3rd *B*	3rd *C*	3rd *B*	3rd *D*	3rd *C*	3rd *B*	3rd *C*	3rd *C*	3rd *B*	3rd *C*	3rd *C*	3rd *B*	3rd *C*
4th *A*	4th *D*	4th *A*	4th *A*	4th *A*	4th *A*	4th *A*	4th *D*	4th *A*	4th *A*	4th *D*	4th *A*	4th *A*	4th *D*	4th *A*	4th *D*	4th *D*	4th *A*	4th *D*

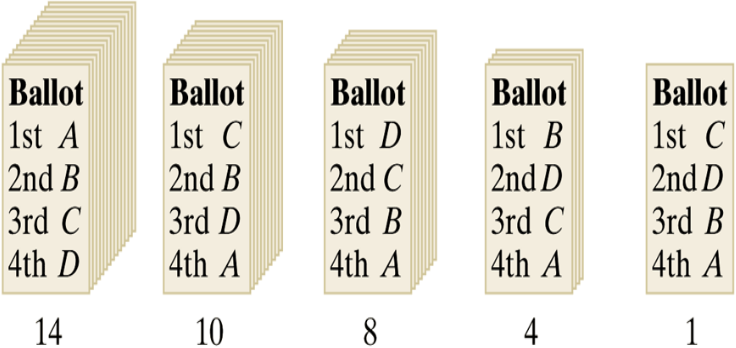
Ballot
1st A
2nd B
3rd C
4th D
14
Ballot
1st C
2nd B
3rd D
4th A
10
Ballot
1st D
2nd C
3rd B
4th A
8
Ballot
1st B
2nd D
3rd C
4th A
4
Ballot
1st C
2nd D
3rd B
4th A
1

TABLE 1-1 Preference Schedule for the Math Club Election

Number of voters	14	10	8	4	1
1st choice	*A*	*C*	*D*	*B*	*C*
2nd choice	*B*	*B*	*C*	*D*	*D*
3rd choice	*C*	*D*	*B*	*C*	*B*
4th choice	*D*	*A*	*A*	*A*	*A*

Voting Theory Terminology

Preference ballot: A ballot on which each voter ranks all eligible candidates, from first to last place. (Ties are not allowed.)

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Preference schedule: A table of how many times each possible ballot was submitted.

Voting method: A mathematical procedure that uses data from the preference schedule to determine a winner.

The Plurality Method (Tannenbaum, §1.2)

The Plurality Method: Whoever receives more first-place votes than any other candidate wins the election.

(a.k.a. "simple plurality"; "first-past-the-post"; "standard voting")

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- By far the most simple and widely-used voting method
- May require a tiebreaker (which we won't worry about)

The Plurality Method

Example: The Math Club election

# Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

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3rd choice	C	D	B	C	B
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The Plurality Method

Example: The Math Club election

# Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Candidate	Votes
A	14
B	4
C	11
D	8

The Plurality Method

Example: The Math Club election

# Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Candidate	Votes
A	**14**
B	4
C	11
D	8

- **Candidate A is declared the winner.**

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An important distinction:

- **Majority** means **"more than 50% of the votes"**
- **Plurality** just means **"more votes than any other candidate"**

These terms are not synonyms!

Majority vs. Plurality

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Every majority is a plurality,
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In every election, some candidate receives a plurality,
but there need not be a majority candidate.

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Majority vs. Plurality

Example 1: Math Club election.

- 37 votes cast; majority $= 19$ votes (since $37/2 = 18\frac{1}{2}$).
- Results: Alisha 14, Boris 4, Carmen 11, Dave 8.
- Alisha received a plurality, but not a majority.

Example 2: 1998 Minnesota gubernatorial election.

- Majority $=$ any percentage above 50%.
- Results: Ventura 37%, Coleman 34%, Humphrey 28%.
- Ventura received a plurality, but not a majority.

Evaluating the Plurality Method

The Plurality Method: Count the first-place votes received by each candidate. Whoever receives the most first-place votes is declared the winner.

- **If there is a majority candidate**, the Plurality Method selects that candidate as the winner (because every majority is also a plurality).
- **If there is no majority candidate**, then the Plurality Method may produce problematic results.

To see why, we need to look at the whole preference schedule.

Evaluating the Plurality Method

Example: The Math Club election

# Voters	**14**	**10**	**8**	**4**	**1**
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Evaluating the Plurality Method

Example: The Math Club election

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3rd choice	C	D	B	C	B
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Candidate	**Votes**
A	**14**
B	4
C	11
D	8

Evaluating the Plurality Method

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Candidate	**Votes**
A	**14**
B	4
C	11
D	8

- Candidate A is declared the winner...

Evaluating the Plurality Method

Example: The Math Club election

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1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	**D**	**A**	**A**	**A**	**A**

Candidate	**Votes**
A	**14**
B	4
C	11
D	8

- Candidate A is declared the winner...
- despite being the **last choice** of a **majority** of voters!

Evaluating the Plurality Method

Example: The Math Club election

# Voters	**14**	**10**	**8**	**4**	**1**
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	**D**	**A**	**A**	**A**	**A**

Candidate	**Votes**
A	**14**
B	4
C	11
D	8

- Candidate A is declared the winner. . . ★
- despite being the **last choice** of a **majority** of voters!
- **Any other candidate would beat A in a head-to-head election!**

Fairness Criteria

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Note: "Satisfies" means "**always** satisfies." That is, if there is a majority candidate, then that candidate is **always** declared the winner by the Plurality Method.

The Condorcet Criterion

The Condorcet Criterion: If Candidate Z would beat any other candidate in a head-to-head contest, then Candidate Z should win the election.

(Such a candidate Z, if one exists, is called a **Condorcet candidate** or **Condorcet winner**. Not every election necessarily has one.)

Back to the Math Club Election

# Voters	**14**	**10**	**8**	**4**	**1**
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Is there a Condorcet winner?

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The Condorcet Criterion

The Condorcet Criterion: If Candidate Z would beat any other candidate in a head-to-head contest, then Candidate Z should win the election.

- The Plurality Method **fails** the Condorcet Criterion.
- That is, it is possible for an election to have a Condorcet candidate, but for that candidate not to win under the Plurality Method.
- (This doesn't mean that a Condorcet candidate **never** wins — only that s/he **might not** win.)

The Plurality Method and Fairness Criteria

The Plurality Method **satisfies** the Majority Criterion.

- That is, in **every** election held using the Plurality Method, if there is a majority candidate, then that candidate will win.

The Plurality Method **fails** the Condorcet Criterion.

- That is, in **some** elections held using the Plurality Method, there is a candidate who would beat every other candidate head-to-head, but does not win the election.

The Plurality Method and Strategic Voting

The Plurality Method is often vulnerable to strategic voting:

> *"If my favorite candidate has no chance to win, then maybe I should vote instead for someone I like less, but who has a chance to win."*

The Plurality Method and Strategic Voting

Example: The KU Tiddlywinks Club is trying to decide what kind of pizza to order — sausage, eggplant, or pineapple. The club members' preferences are as follows:

# Voters	**5**	**3**	**7**	**2**
1st choice	S	S	E	P
2nd choice	E	P	P	E
3rd choice	P	E	S	S

1. Who wins?

2. If you are one of the two voters who loves pineapple and hates sausage, what should you do?

The Plurality Method and Strategic Voting

Example: The KU Tiddlywinks Club is trying to decide what kind of pizza to order — sausage, eggplant, or pineapple.

	Real preferences				**Actual vote**		
# Voters	**5**	**3**	**7**	**2**	**5**	**3**	**9**
1st choice	S	S	E	P	**S**	**S**	**E**
2nd choice	E	P	P	E	**E**	**P**	**P**
3rd choice	P	E	S	S	**P**	**E**	**S**

Plurality winner: Sausage
Plurality winner: Eggplant

The Plurality Method and Strategic Voting

- Whenever voters have an incentive to **vote strategically** (that is, to vote differently from their true preferences), the **voting method may be flawed.**

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- **Reminder:** Mathematics takes no position on whether strategic voting is moral or immoral.
- Strategic voting is impossible to eliminate entirely.
- On the other hand, strategic voting **reduces the effectiveness** of the voting method.
- So, in order to reflect society's preferences as accurately as possible, mathematics should try to **minimize the opportunities for strategic voting.**