Probabilities and Yarboroughs

Question 1: What is the probability of being dealt a Yarborough?

As so often in mathematics, in order to answer the original question, we start by asking a different question.

Question 2: How many possible Yarboroughs are there?

If we know this, then we can divide the answer by the number of possible bridge hands (which we now know is 635013559600) to get the answer to the original Question 1.

On the other hand, Question 2 can be answered by the same techniques that we used to count all possible hands. What is a Yarborough, after all? It's a hand with no card higher than a nine. That is, it is a hand of thirteen cards selected from a set of thirty-two cards (namely the deuce through nine of each suit: 8 cards per suit $\times$ 4 suits = 32 cards.) Therefore,

$$\textbf{number of possible Yarboroughs} = \binom{32}{13} = \frac{32!}{13! \times 19!} = \mathbf{347373600.}$$

That takes care of Question 1. As for Question 2, the answer is

$$\textbf{probability of being dealt a Yarborough} = \frac{\textbf{number of possible Yarboroughs}}{\textbf{number of possible hands}}$$

$$= \binom{32}{13} \Big/ \binom{52}{13} = \frac{\mathbf{347373600}}{\mathbf{635013559600}} \approx \mathbf{0.00054703 = 0.054703\%}$$

or about 1 in 1828. So you can expect to be dealt a Yarborough about one of every 1828 hands you play. But that doesn't mean you can't have four Yarboroughs in a row, or go fifteen years without being dealt a Yarborough (lucky you!). It just means that *on average*, one out of every 1828 or so hands you get will be a Yarborough.

Question 3: What is the probability of being dealt at least one Yarborough in a 24-board session?

The naive answer is to take the number 0.00054703 and multiply it by 24. Actually, that's not mathematically precise (although you do happen, in this case to get a number that is reasonably close to the correct answer). Here is an analogy. You flip a coin. The chance that it comes up heads is 50%. Does that mean that if you flip it twice, the chance of getting at least one heads is 2×50% = 100%? Of course not! The chance of getting tails twice is 25%, so the chance of that *not* happening — that is, of getting heads at least once — is 75%.

The same reasoning can be used to answer Question 3. The chance of *not* being dealt a Yarborough on any particular deal is approximately

$$1 - 0.00054703 = 0.99945297$$

and so the chance of not being dealt a Yarborough on any of 24 separate deals is approximately

$$(0.99945297)^{24} \approx 0.9869535$$

and so the chance of being dealt a Yarborough (i.e., not not being dealt a Yarborough) on at least one of 24 deals is approximately

$$1 - 0.9869535 = 0.0130465 = 1.30465\%$$

or about 1 in 77.

Question 4: What is the probability of being dealt at least one Yarborough in each of two 24-board sessions?

(This was Virginia Seaver's question that led to this whole project!)

Using the answer to Question 3, and the same logic, the answer is

$$(0.0130465)^2 \approx 0.0001702122 = 0.01702122\%$$

or about 1 in 5875. Hardly an everyday occurrence, but not that much less frequent than being dealt a Yarborough in the first place (which, remember, was about 1/1828, or roughly three times 1/5875.)