Mathematics in Industry Careers

at the University of Kansas

Kevin Li,
Officer, Model Risk Program Manager
Federal Home Loan Bank of Topeka
Kevin.Li@FHLBTopeka.com
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WHO WE ARE

- Federal Home Loan Bank of Topeka (FHLBank) is a federally chartered corporation under the authority of the Federal Home Loan Bank Act of 1932;

- Our primary business is making collateralized loans and providing other banking services to our member institutions (banks, thrifts, credit unions and insurance companies);

- We are a cooperative owned by our nearly 700 member institutions located across our four-state district (Colorado, Kansas, Nebraska and Oklahoma);

- FHLBank employs approximately 240 people and is located in Topeka, Kansas.

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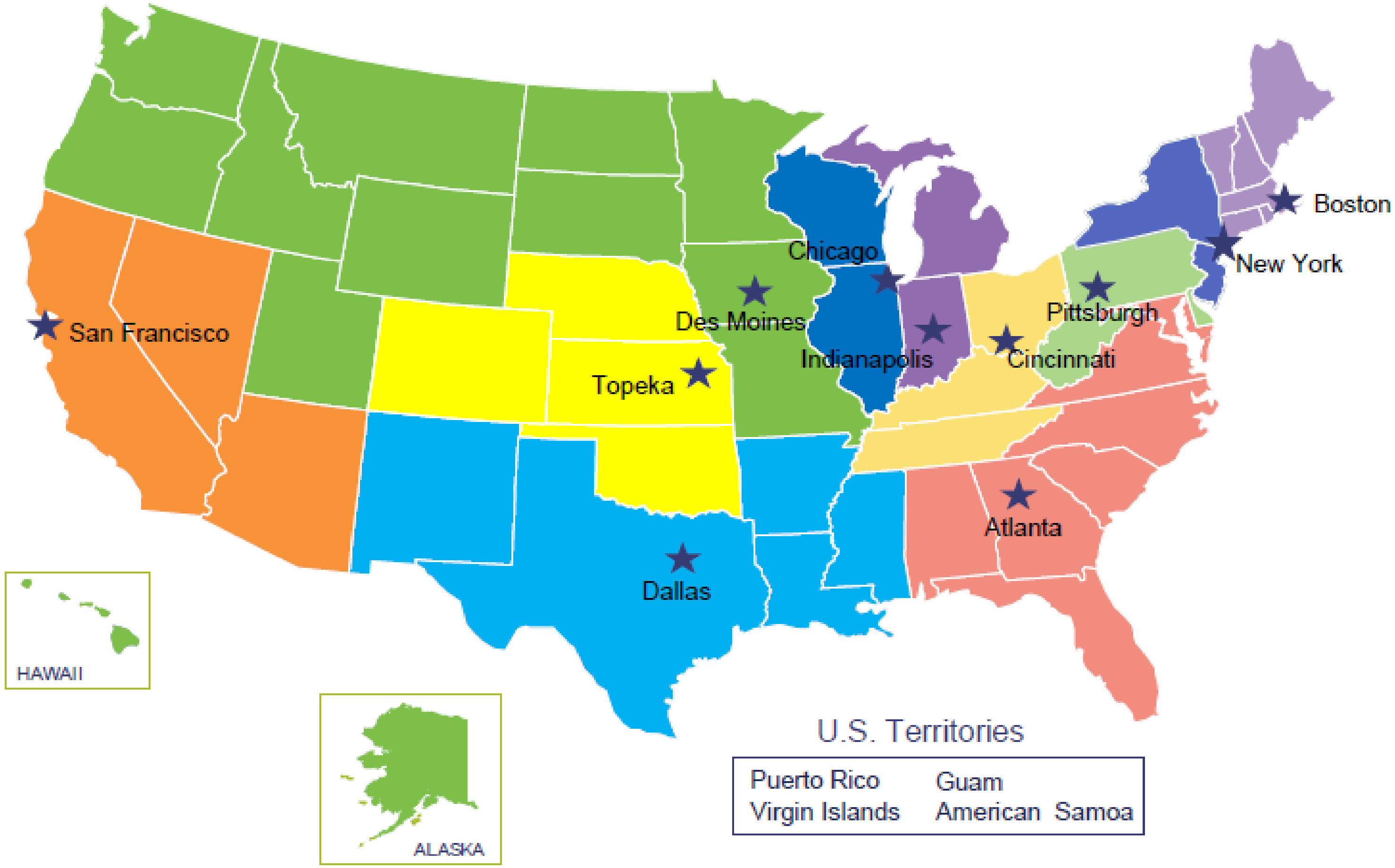
FHLBANK SYSTEM OVERVIEW

- FHLBank Topeka is one of 11 FHLBanks



WHAT WE USE ADVANCED MATH FOR

- **Capital Markets**: Pre-trading Analysis, Margin Calculation (SIMM Model), Fund Transfer Pricing (net interest margin analysis), Hedging Strategy, Portfolio Optimizing;

- **Market Risk**: Interest Rate Projection (SDE e.g., BGM, Hull White, BDT etc.), Market Valuation (Black Model, Lattice Tree Model, Monte Carlo Simulation), Risk Measuring (duration, convexity and other Greeks), Scenario Analysis, Balance Sheet Management/Simulation and Horizon Valuation;

- **Credit Risk**: Credit Rating, Probability of Default, Mortgage Modeling (Prepayment Model, State Transition, Loss Given Default, Cure Rate Models etc.), Time Series, Regression, Sampling etc.;

- **Model Risk**: Model Validation, Model Performance Monitoring (benchmarking, back-testing, sensitivity tests, stress testing, attribution analysis), Model Governance, Risk Controls and Reporting.

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EXAMPLES OF MODELS – Hull White

- **Use and Assumptions:** to model future interest rate with assumptions that the market has no-arbitrage, Short rates have a normal distribution and are subject to mean reversion, bond and derivative pricing

- **Mathematical Form**: The model can be described by Stochastic Deferential Equation(s) as follows:
  - One-factor model: $dr(t) = [\theta(t) - \alpha(t)r(t)]\,dt + \sigma(t)\,dW(t).$
  - Two-factor model: $d\,f(r(t)) = [\theta(t) + u - \alpha(t)\,f(r(t))]\,dt + \sigma_1(t)\,dW_1(t),\, du = -bu\,dt + \sigma_2\,dW_2(t)$

- **Other Interest Rate Models**: BDT: $d\ln(r) = [\theta_t + \frac{\sigma'_t}{\sigma_t}\ln(r)]dt + \sigma_t\,dW_t$

  Black–Scholes (for option valuation): $\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$

  Brace Gatarek Musiela (BGM also known as LIBOR market model):

$dL_j(t) = \mu_j(t)L_j(t)dt + \sigma_j(t)L_j(t)dW^{Q_{T_{j+1}}}(t)$ $L_j,\, j = 1, \ldots, n$ as lognormal processes

Implementation Approaches: Monte-Carlo simulation, trees or lattice approach – *please note these models have been implemented in an integrated vendor application/system purchased by the FHLBank but we need to understand the general concepts and modeling processes while using these models for analyses.*

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EXAMPLES OF MODELS – Nelson Siegel

- **Goal**: To parameterize an interest rate curve in the following Polynomial form

$$y(m) = a_0 + a_1 m + a_2\, m^2,$$

where m is the maturity and a_0, a_1, a_2 are parameters to be fitted and $y(m)$ is the interest curve at maturity year m.

- **NS Function**: $$y(m) = \beta_0 + \beta_1 \frac{[1 - \exp(-m/\tau)]}{m/\tau} + \beta_2 \left(\frac{[1 - \exp(-m/\tau)]}{m/\tau} - \exp(-m/\tau) \right)$$

where $\beta_0, \beta_1, \beta_2$= long term, short term and medium-term rates and τ= decay factor (see Diebold and Li [2006]; Bolder and Stréliski [1999])

- **Determine the Parameters**: fit the curve using historical data via regression analysis such as least-squares or a similar algorithm

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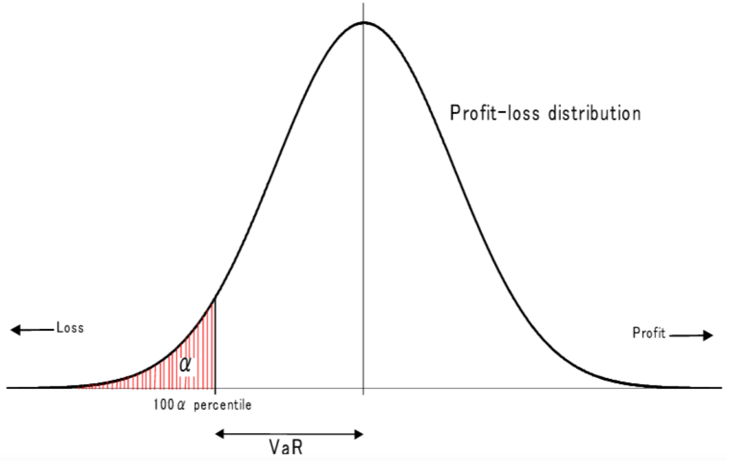
EXAMPLES OF MODELS – Value-at-Risk (VaR)

- **Goal**: Value at risk (VaR) is a measure of the risk of loss for investments
- **Mathematical Definition**: p-VaR is defined such that the probability of a loss greater than VaR is (at most) p while the probability of a loss less than VaR is (at least) 1−p. Or alternatively at confidence 1-p, the loss will be less than VaR. Let X denote the financial loss and $\alpha \in (0,1)$, then the definition can be express as:

$$\mathrm{VaR}_{1-\alpha}(X) := \inf_{t \in \mathbb{R}} \{t : \Pr(X \leq t) \geq 1 - \alpha\},$$

- **Methods**: Parametric vs. Non-Parametric VaR
  - Parametric VaR: uses mean-variance analysis to predict future outcomes based on past experience; straightforward, but makes the assumption that possible outcomes are normally distributed about the mean
  - Non-parametric VaR: few assumptions but less powerful than parametric statistics



- **Weakness/Limitations**: false sense of security, only as good as the inputs and assumptions. However, VaR can be useful, as long as you keep its weaknesses in mind and don't take VAR for something it isn't.

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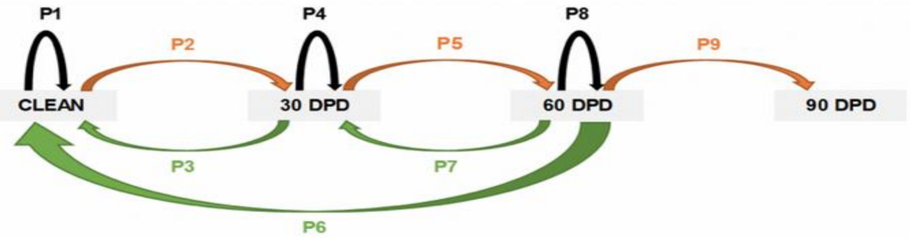
EXAMPLES OF MODELS – Mortgage Modeling

- **Prepay Model**: To estimate the level of early payoffs on a loan or group of loans in a set period of time given possible changes in interest rates
- **Transition Model:** To model a loan level state transition with the probability of transiting to a new state, which can be modeled by a Markovian Chain with a matrix as follows:

$$P_{ij}(t) = \begin{bmatrix} p_{c,c}(t) & p_{c,30}(t) & p_{c,60}(t) & p_{c,90}(t) & p_{c,p}(t) & p_{c,d}(t) \\ p_{30,c}(t) & p_{30,30}(t) & p_{30,60}(t) & p_{30,90}(t) & p_{30,p}(t) & p_{30,d}(t) \\ p_{60,c}(t) & p_{60,30}(t) & p_{60,60}(t) & p_{60,90}(t) & p_{60,p}(t) & p_{60,d}(t) \\ p_{90,c}(t) & p_{90,30}(t) & p_{90,60}(t) & p_{90,90}(t) & p_{90,p}(t) & p_{90,d}(t) \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



- **Probability of default (PD) Model**: describing the likelihood of a default over a particular time horizon. It provides an estimate of the likelihood that a borrower will be unable to repay the debt
- **Loss given default (LGD) Model**: the loss amount if a borrower defaults

All these models can be implemented via regression using historical data for model fitting to determine the required parameters

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Other Mathematical/Statistical Tools

- **Logistic model**: To estimate the relationships between different components of market information (interest rate, volatility, etc.)

$$\text{Log}\frac{v}{1-v} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

where $v =$ the volatility of the interest rate move, $x_1 =$ interest rate with 2yr maturity, $x_2 =$ interest rate with 10yr maturity

- **Cubic Interpolation**: for example, interpolation between p_0 and p_1
- **Optimization, Sampling, Score-card Rating, etc.**

$$\boldsymbol{p}(t) = (2t^3 - 3t^2 + 1)\boldsymbol{p}_0 + (t^3 - 2t^2 + t)\boldsymbol{m}_0 + (-2t^3 + 3t^2)\boldsymbol{p}_1 + (t^3 - t^2)\boldsymbol{m}_1,$$

- **Machine Learning/Artificial Intelligence (not currently used but likely used in the future)**

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THANK YOU!!

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